Rec 1

Monday, October 2, 2023 01:07

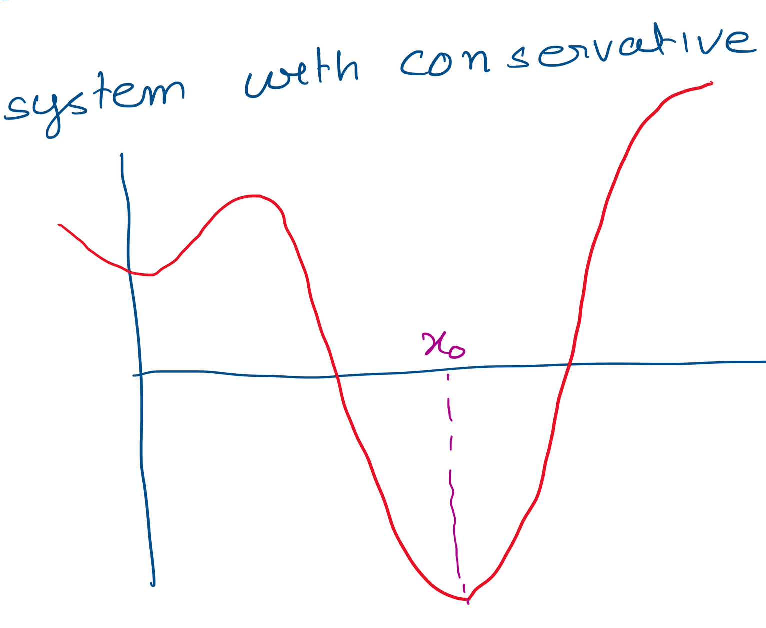
You have learnt a lot about SHOs this week. One of the reasons we focus on this topic is because its one of the few things we know how to solve.

" The career of a young theoretical physicist is treating HO in increasing levels of abstraction "
— Sydney Coleman

So, instead of learning new laws, we will focus on applying what we know to increasingly complex systems.

Lets start with understanding where SHOs occur.

For any system with conservative forces



$$U(x) = U(x_0) + \left.\frac{dU}{dx}\right|_{x_0}(x-x_0) + \frac{1}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2 + \dots$$

(taylor expanded about equilibrium)

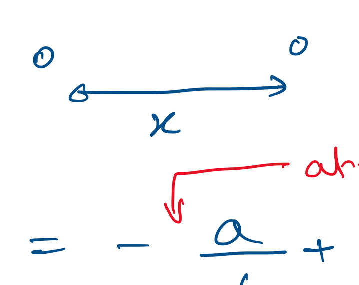
Now, using $F = -\frac{dU}{dx}$

$$F = 0 - \underbrace{\left.\frac{dU}{dx}\right|_{x=x_0}}_{0} - \left.\frac{d^2U}{dx^2}\right|_{x=x_0}(x-x_0) -$$

$$\therefore\ F = -\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)$$

$$\& \ U = U(x_0) + \frac{1}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2$$

1) Lets apply this to the case of diatomic molecules,



$$U(x) = -\frac{a}{x^6} + \frac{b}{x^{12}}\ ;\ a, b > 0$$

attractive force

repulsive forces

i) is this SHO?

Not yet

ii) Find the minima

$$F(x) = 0 = -\frac{6a}{x_0^7} + \frac{12b}{x_0^{13}}$$

$$\Rightarrow x_0 = (2b/a)^{1/6}$$

iii) Find the eqns of motion

$$F(x) = -\frac{6a}{(x_0+\Delta x)^7} + \frac{12b}{(x_0+\Delta x)^{13}}$$

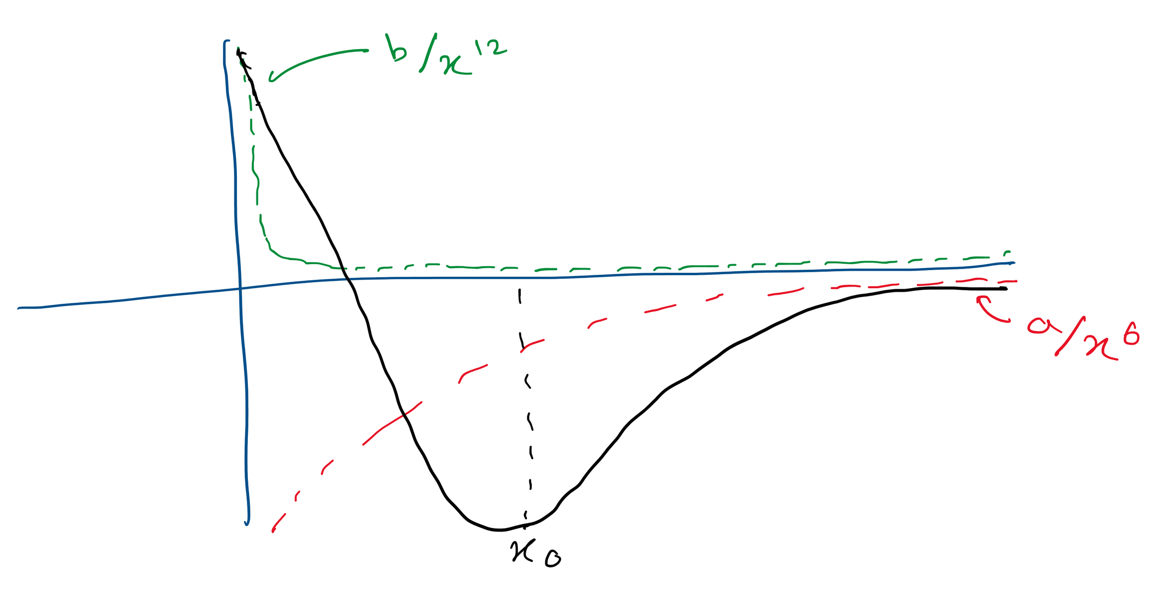
$$= -\frac{6a}{x_0^7}\left(1+\frac{\Delta x}{x_0}\right)^{-7} + \frac{12b}{x_0^{13}}\left(1+\frac{\Delta x}{x_0}\right)^{-13}$$

are equal

$$= \alpha\left[-1 + \frac{7\Delta x}{x_0} + 1 - 13\frac{\Delta x}{x_0}\right]$$

$$= -\frac{6\alpha}{x_0}\Delta x$$

$$= -\frac{36a}{x_0^8}\Delta x$$



2) Lets now solve the eqns of motion using conservation of energy,

$$E = \frac{1}{2}mv^2 + mgl(1-\cos\theta)$$

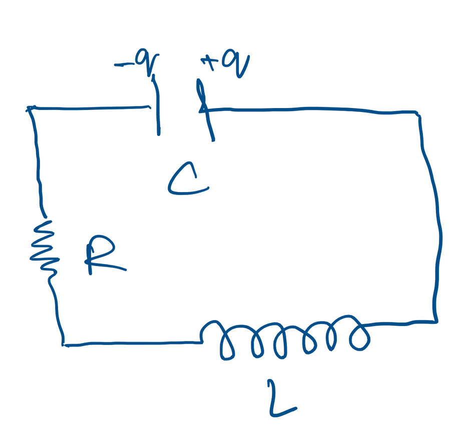
$$\Rightarrow E = \frac{1}{2}ml^2\dot{\theta}^2 + mgl(1-\cos\theta)$$

$$\Rightarrow \frac{dE}{dt} = 0 = ml^2\dot{\theta}\ddot{\theta} + mgl\sin\theta\,\dot{\theta}$$

Using small angle approximation,

$$\ddot{\theta} = -g/l\ \theta$$

3) Lets make life a bit more complicated.



$$\frac{q}{c} - L\frac{dI}{dt} - IR = 0$$

$$\Rightarrow L\frac{dI}{dt} - q/c + IR = 0$$

$$\boxed{I = -dq/dt}$$

$$\Rightarrow L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{c} = 0$$

$$\Rightarrow \ddot{q} + \frac{R}{L}\dot{q} + \frac{q}{LC} = 0$$

Lets solve the generic eqn instead.

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = 0$$

Ansatz $\rightarrow$ $z(t) = Ce^{\alpha t}$

$$\alpha^2 + \gamma\alpha + \omega_0^2 z = 0$$

$$\alpha = \frac{-\gamma \pm \sqrt{\gamma^2 - 4\omega_0^2}}{2}$$

$$\therefore\ z(t) = C_1 e^{\alpha_1 t} + C_2 e^{\alpha_2 t}$$

$$= e^{-\gamma/2\, t}\left[C_1 e^{+\frac{\sqrt{\gamma^2-4\omega_0^2}}{2}t} + C_2 e^{-\sqrt{\gamma^2-4\omega_0^2}/2\ t}\right]$$

always dies out

4) How about a driven oscillator,

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = F_0/m\cos(\omega t)$$

so how do you find the right ansatz

$$\ddot{y} - 4\dot{y} - 12y = 3e^{st}$$

Ansatz $\rightarrow$ Ae^{st}

$$\ddot{y} - 4\dot{y} - 12y = 2t^3 - t^2$$

Ansatz $\rightarrow$ $At^3 + Bt^2 + Ct + D$

$\therefore$ for

$$\ddot{z} + \gamma\dot{z} + \omega_0^2 z = F_0/m\ e^{i\omega t}$$

$$\therefore\ \underbrace{z_p(t)}_{\text{ansatz}} = Ae^{i(\omega t - \delta)}$$

$$\therefore\ A e^{i(\omega t-\delta)}\left[-\omega^2 + i\gamma\omega + \omega_0^2\right] = F_0/m\ e^{i\omega t}$$

$$(\omega_0^2 - \omega^2)A + i\gamma\omega A = F_0/m\ e^{i\delta}$$

$$F_0/m\ \sin\delta = \gamma\omega A$$

$$F_0/m\ \cos\delta = (\omega_0^2 - \omega^2)A$$

$$\therefore\ A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2-\omega^2)^2 + \gamma^2\omega^2}} \qquad \delta = \tan^{-1}\left(\frac{\gamma\omega}{\omega_0^2 - \omega^2}\right)$$