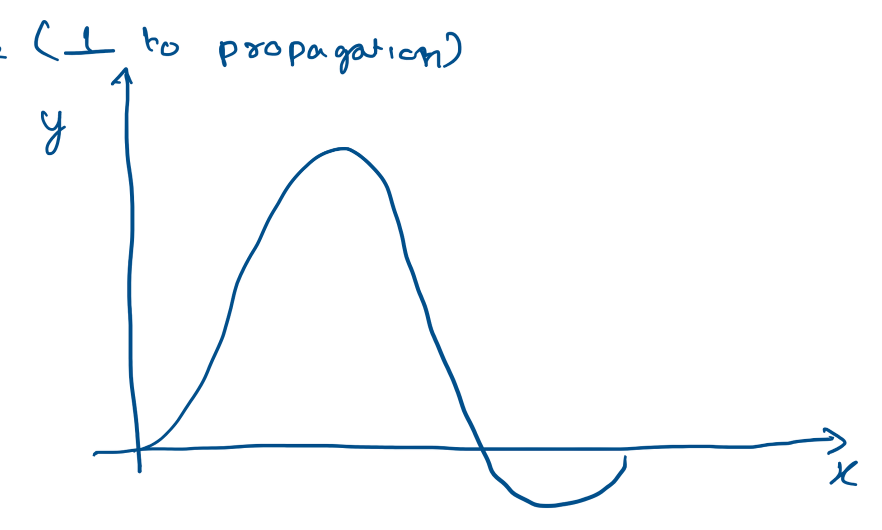
Lecture 9

Tuesday, November 1, 2022 11:02 PM

Waves

Large no of coupled oscillators that propagates some disturbance.
↳ cause of propagation

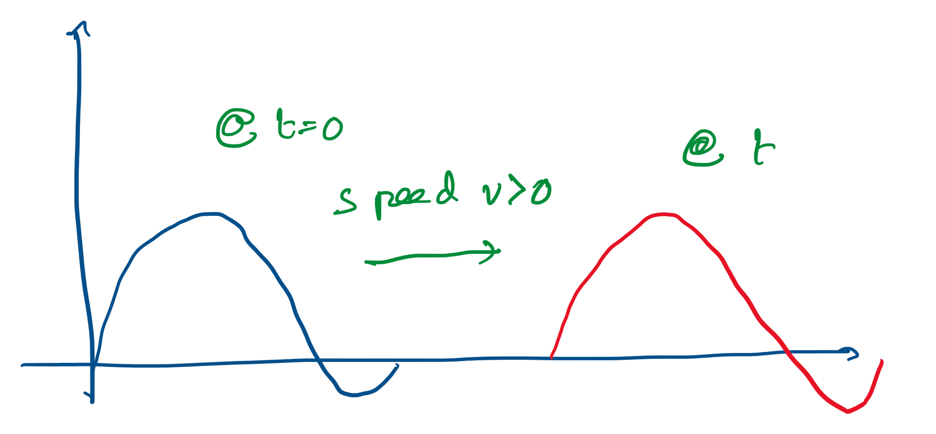
Transverse ($\perp$ to propagation)



$y(x,t)$
↑ generic coordinate → can be $\vec{E}, \vec{B}, \vec{x}$ anything.

Longitudinal (along the direction of propagation e.g. sound)

So what is the most generic idea of a wave.?



$\underbrace{f(x-vt)}_{\text{forward propagating wave}} + \underbrace{g(x+vt)}_{\text{backward propagating wave.}}$ ⟵ What is this a soln. to?

Speed of wave → $\dfrac{\text{coefficient of } t}{\text{coefficient of } x}$

N – coupled Oscillators

$\left.\begin{matrix} N \to \infty \\ \Delta x \to 0 \end{matrix}\right\} \underbrace{N\Delta x \to \text{finite}}_{\text{we look at collective behavior}}$

↓ eqs of motion

$$\boxed{\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}}$$ → For this course this is enough

$\Rightarrow \ddot{y} = v^2 y''$ e.g. $\ddot{\vec{E}} = \frac{1}{\mu_0 \varepsilon_0} \vec{E}''$ ⟵ emerges from Maxwells equation

$c = \sqrt{1/\mu_0\varepsilon_0}$

Plane wave solns ⟵ amplitude is independent of position & is sinusoidal

$y(x,t) \sim \cos(kx \pm \omega t) \sim Ae^{i(kx \pm \omega t)}$
A ⟵ doesn't depend on x
↑ have dependence on other types

We try to find for ω & k that solves wave equation.

& then we find $\omega = \omega(k)$ ⟵ Dispersion equation.

e.g. $\ddot{y} = (-i\omega)^2 y$
$y'' = (ik)^2 y$

$\Rightarrow \ddot{y} = v^2 y''$

$-\omega^2 y = -v^2 k^2 y$

$\Rightarrow v = \omega/k$
↑ property of material, type of wave, etc.

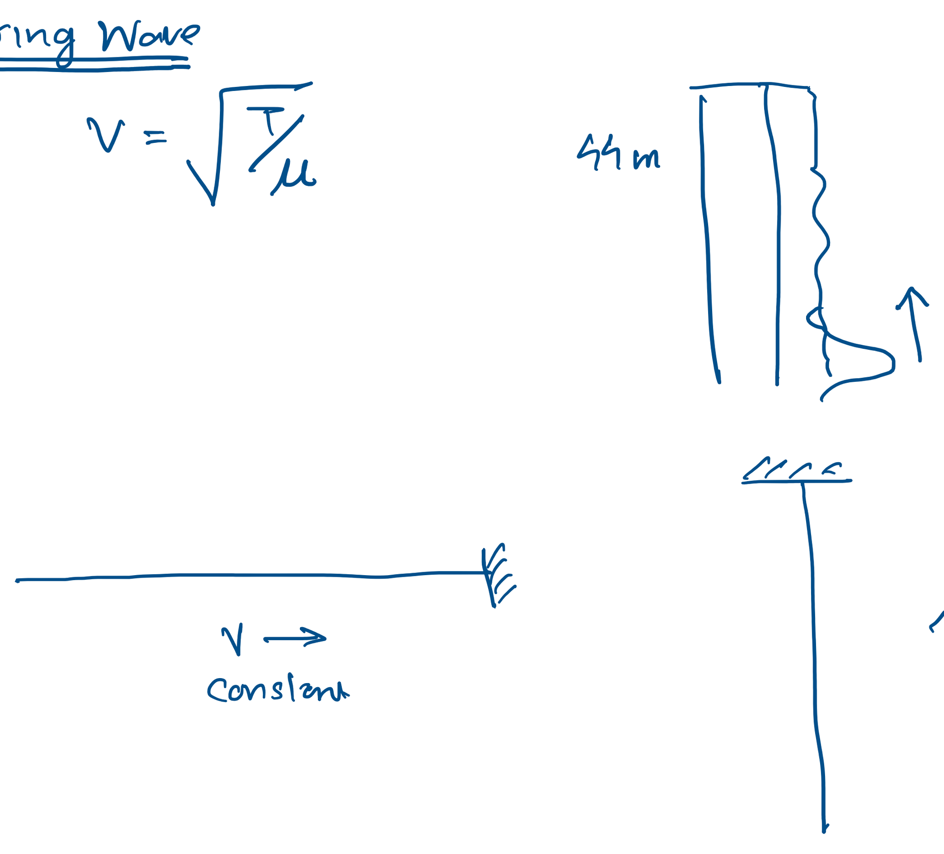
$\omega = vk$ ⟵ linear wave equation

ω → source dependent, medium independent
v, k → medium dependent.
↑ wave number $= 2\pi/\lambda$

$\Rightarrow v = \nu\lambda$

String Wave

$v = \sqrt{T/\mu}$



→ How long to reach top?

↑ v increases upwards

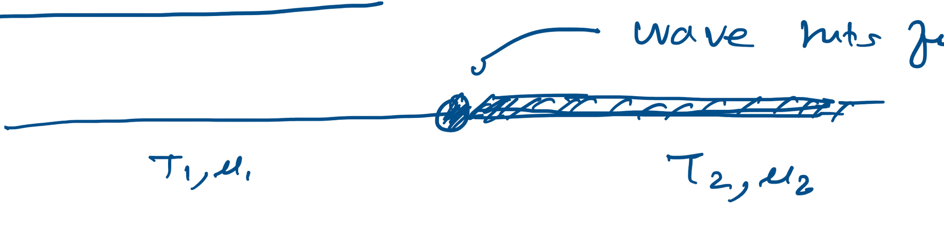
$T = \mu h g$

Time to go up a tower of 44m

$v = \sqrt{hg}$

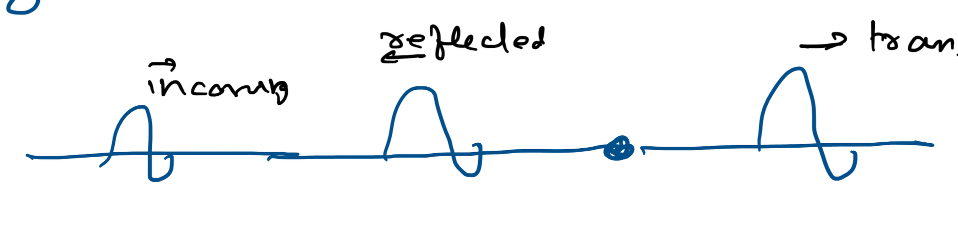
$t = \int_0^{44} \frac{dh}{\sqrt{hg}}$

Waves @ boundaries



$$y(x,t) = \begin{cases} y_L(x,t), & x \leq 0 \\ y_R(x,t), & x \geq 0 \end{cases}$$

∴ for $t > 0$



Left → $\ddot{y}_L = \frac{T_1}{\mu_1} y_L''$ Right → $\ddot{y}_R = \frac{T_2}{\mu_2} y_R''$

At junction, there must be continuity

$\left.\begin{matrix} y_L(0,t) = y_R(0,t) \\ \Rightarrow y_i(0,t) + y_R(0,t) = y_T(0,t) \end{matrix}\right\}$ dirichlet boundary condition

Force balance at $x=0$ (as it is a massless point)

$T_1 \left.\frac{\partial y_L}{\partial x}\right|_0 = T_2 \left.\frac{\partial y_R}{\partial x}\right|_0$ } Neumann condition
↑ angle of tension

Once you work this out,

$y_r = \overbrace{\left(\frac{z_1 - z_2}{z_1 + z_2}\right)}^{r} y_i$ where $\left.\begin{matrix} z_1 = \frac{T_1}{v_1} = \sqrt{T_1 \mu_1} \\ z_2 = T_2/v_2 = \sqrt{T_2 \mu_2} \end{matrix}\right\}$ impedance

$y_t = \underbrace{\frac{2z_1}{z_1 + z_2}}_{t} y_i$

shapes dont change

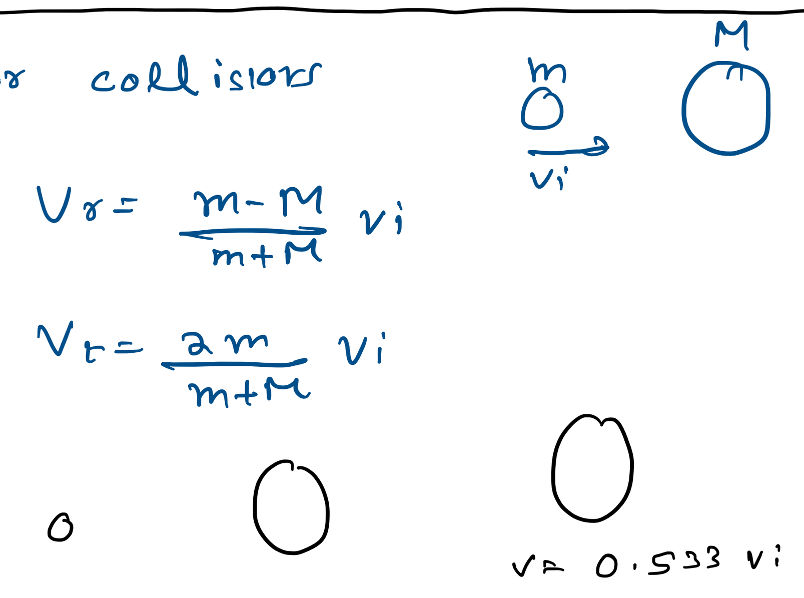
If $z_2 > z_1$

$y_r = \underset{<0}{r}\, y_i$ ⟵ wave flips sign

If $z_1 > z_2$

$y_t = \underset{>1}{t}\, y_i$ ⟵ there is amplification

for collisions



$V_r = \frac{m - M}{m + M} v_i$

$V_t = \frac{2m}{m + M} v_i$

$v = 0.533\, v_i$

Impedance Matching

This holds for all sorts of physical events.