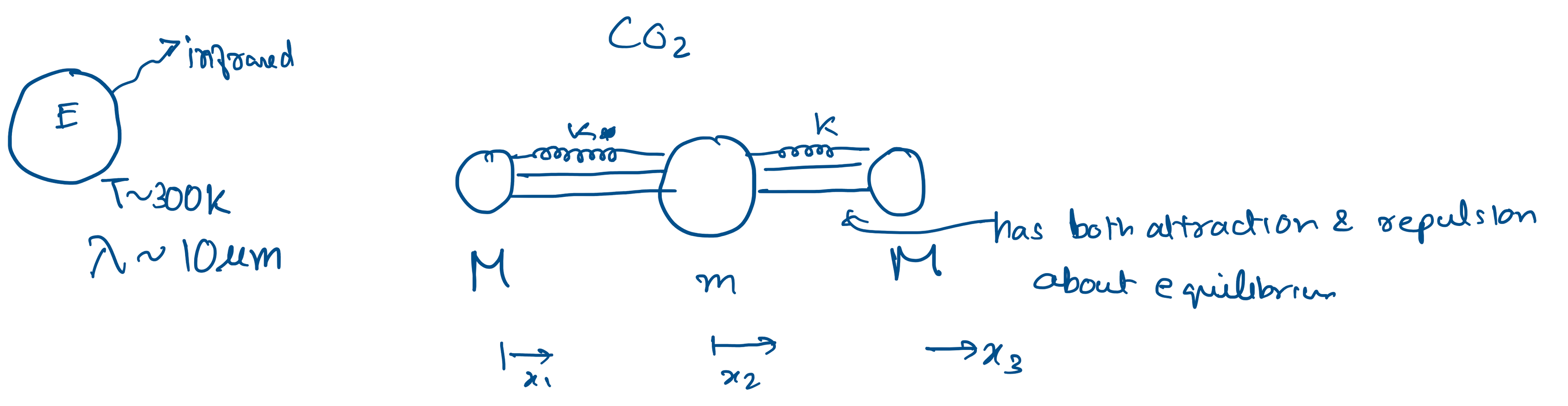
Lecture 8

Friday, October 21, 2022 12:43 AM

CO_2



$$M\ddot{x}_1 = -k(x_1 - x_2)$$
$$m\ddot{x}_2 = +k(x_1 - x_2) - k(x_2 - x_3)$$
$$M\ddot{x}_3 = +k(x_2 - x_3)$$

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} = k \begin{pmatrix} -\frac{1}{M} & \frac{1}{M} & 0 \\ \frac{1}{m} & -\frac{2}{m} & \frac{1}{m} \\ 0 & \frac{1}{M} & -\frac{1}{M} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

$$\ddot{X} = -MX$$

Normal mode frequencies are eigenvalues of M (ω^2)

In reality 10^{13} Hz $\left\{ \begin{array}{l} \omega_1^2 = k/M \rightarrow \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \\ \omega_2^2 = \frac{k(m+2M)}{mM} \rightarrow \begin{pmatrix} 1 \\ -2M/m \\ 1 \end{pmatrix} \end{array} \right.$

$\lambda = \frac{c}{f} = 10\mu m$

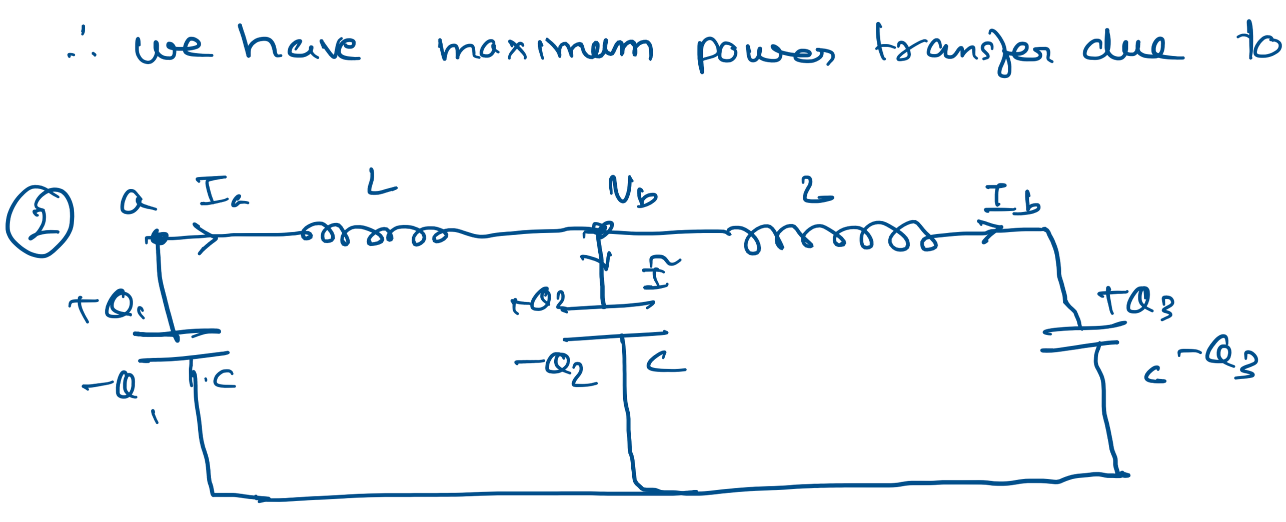
$$\omega_3^2 = 0 \rightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

translation (moving in sync)

$$\lambda = \frac{c}{f} \sim 10\mu m$$

$\therefore$ we have maximum power transfer due to resonance.

(2)



How many modes? (2)

$\tilde{I} = I_a - I_b$

$$\cancel{V_a} - L\dot{I}_a - \frac{Q_2}{C} + \frac{Q_1}{C} = \cancel{V_a} \Rightarrow LC\dot{I}_a = Q_1 - Q_2$$

$$V_b - L\dot{I}_b - \frac{Q_3}{C} + \frac{Q_2}{2} = 0 \Rightarrow LC\dot{I}_b = Q_2 - Q_3$$

$$\Rightarrow LC\ddot{I}_a = -I_a - \tilde{I}$$
$$LC\ddot{I}_b = \tilde{I} - I_b$$

$\dot{Q}_1 = -I_a$

$\dot{Q}_2 = \tilde{I}$

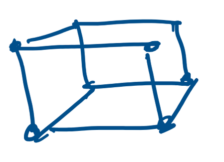
$\dot{Q}_3 = I_b$

$$\Rightarrow LC\ddot{I}_a = -2I_a + I_b$$
$$LC\ddot{I}_b = I_a - 2I_b$$

$$\begin{pmatrix} \ddot{I}_a \\ \ddot{I}_b \end{pmatrix} = \begin{pmatrix} -\frac{2}{LC} & \frac{1}{LC} \\ \frac{1}{LC} & -\frac{2}{LC} \end{pmatrix} \begin{pmatrix} I_a \\ I_b \end{pmatrix} \Rightarrow \begin{array}{l} \omega_1^2 = 3/LC \rightarrow \begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ \omega_2^2 = 1/LC \rightarrow \begin{pmatrix} 1 \\ -1 \end{pmatrix} \end{array}$$

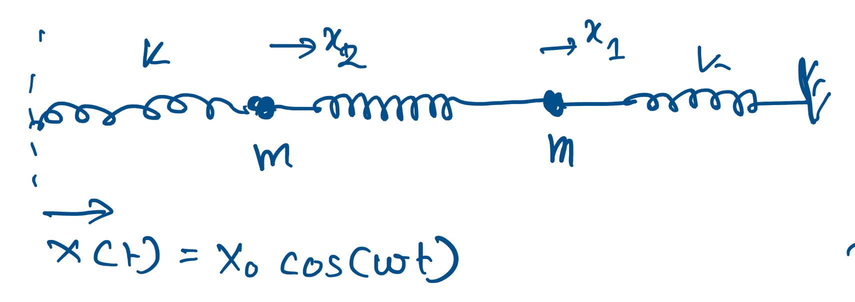
$$I_a(t) = A_1 \cos(\omega_1 t + \phi_1) + B_1 \cos(\omega_2 t + \phi_2)$$
$$I_b(t) = A_1 \cos(\omega_1 t + \phi_1) - B_1 \cos(\omega_2 t + \phi_2)$$

(3) How many normal modes in a cube?



3N modes
8 particles
so 24 modes

(4) Forced Coupled Oscillations



$X > x_2 > x_1$

$$m\ddot{x}_1 = k(x_2 - x_1) - kx_1$$
$$m\ddot{x}_2 = k(X - x_2) - k(x_2 - x_1)$$

$$\ddot{x}_1 = -\frac{2k}{m}x_1 + \frac{k}{m}x_2$$

$$\ddot{x}_2 = \frac{k}{m}x_1 - \frac{2k}{m}x_2 + k\cancel{x_0}\cos(\omega t)$$

$$\left.\begin{array}{l} \ddot{x}_1 + \frac{2k}{m}x_1 - \frac{k}{m}x_2 = 0 \\ \ddot{x}_2 - \frac{k}{m}x_1 + \frac{2k}{m}x_2 = \frac{kx_0}{m}\cos(\omega t) \end{array}\right\}$$

I cant write this now as $\ddot{X} = -MX$

$$\begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} \frac{2k}{m} & -\frac{k}{m} \\ -k/m & 2k/m \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{kx_0}{m}\cos\omega t \end{pmatrix}$$

What are normal modes in the steady state?

(Each particle matches the driving frequency)

$$\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{pmatrix} e^{i\omega t}$$

$$-\omega^2 \begin{pmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{pmatrix} + \begin{pmatrix} & \\ & \end{pmatrix} \begin{pmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{kx_0}{m} \end{pmatrix}$$

$$\Rightarrow \underbrace{\begin{pmatrix} 2k/m - \omega^2 & -k/m \\ -k/m & \frac{2k}{m} - \omega^2 \end{pmatrix}}_{Y} \begin{pmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{pmatrix} = \begin{pmatrix} 0 \\ \frac{kx_0}{m} \end{pmatrix}$$

If this was zero Y is singular & det Y = 0

(ω is known) So these are just simultaneous equations

or

$$\begin{pmatrix} \tilde{A}_1 \\ \tilde{A}_2 \end{pmatrix} = Y^{-1} \begin{pmatrix} 0 \\ \frac{kx_0}{m} \end{pmatrix}$$

$$\tilde{A}_1 = \frac{1}{\underbrace{\left(\frac{3k}{m} - \omega^2\right)\left(\frac{k}{m} - \omega^2\right)}} \frac{k^2 x_0}{m}$$

Blow up at natural frequencies

$$\tilde{A}_2 = \frac{(2k/m - \omega^2)}{\left(\frac{3k}{m} - \omega^2\right)\left(\frac{k}{m} - \omega^2\right)} \frac{kx_0}{m}$$