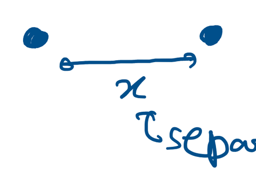
Lecture 2

Thursday, September 29, 2022 10:27 PM

Q1> Diatomic Molecules (H_2)



$$U(x) = \underbrace{-\frac{a}{x^6}}_{\text{attractive force}} + \underbrace{\frac{b}{x^{12}}}_{\text{repulsive forces}} \; ; \; a, b > 0$$

(Comment on presence of attractive & repulsive forces)

i> Is this a SHO?

— not yet. Yes when around local minima

ii> Is there a local minima? Find it.

As the terms in U have opposite signs, they would have a minima

$$F(x) = -\frac{\partial U}{\partial x} = -\frac{dU}{dx} = \underbrace{-\frac{6a}{x^7}}_{\text{attractive}} + \underbrace{\frac{12b}{x^{13}}}_{\text{repulsive}}$$

∴ equilibrium point at

[Always find eq. problems in SHO problems]

$$\frac{6a}{x_0^7} - \frac{12b}{x_0^{13}} = 0$$

$$\Rightarrow x_0^6 = \frac{2b}{a}$$

$$\Rightarrow x_0 = \left(\frac{2b}{a}\right)^{1/6}$$

iii> Show that it leads to a SHO

[perturb from eq. Common recipe for SHO problems]

$$F(x) = -\frac{6a}{(x_0+\Delta x)^7} + \frac{12b}{(x_0+\Delta x)^{13}}$$

$$= -\frac{6a}{x_0^7}\left(1+\frac{\Delta x}{x_0}\right)^{-7} + \frac{12b}{x_0^{13}}\left(1+\frac{\Delta x}{x_0}\right)^{-13}$$

When $\Delta x \ll x_0$, we do binomial expansion & approximate

$$= \underbrace{-\frac{6a}{x_0^7}}_{\alpha}\left(1 - \frac{7\Delta x}{x_0}\right) + \underbrace{\frac{12b}{x_0^{13}}}_{\alpha}\left(1 - \frac{13\Delta x}{x_0}\right)$$

$$= \alpha\left[-1 + \frac{7\Delta x}{x_0} + 1 - \frac{13\Delta x}{x_0}\right]$$

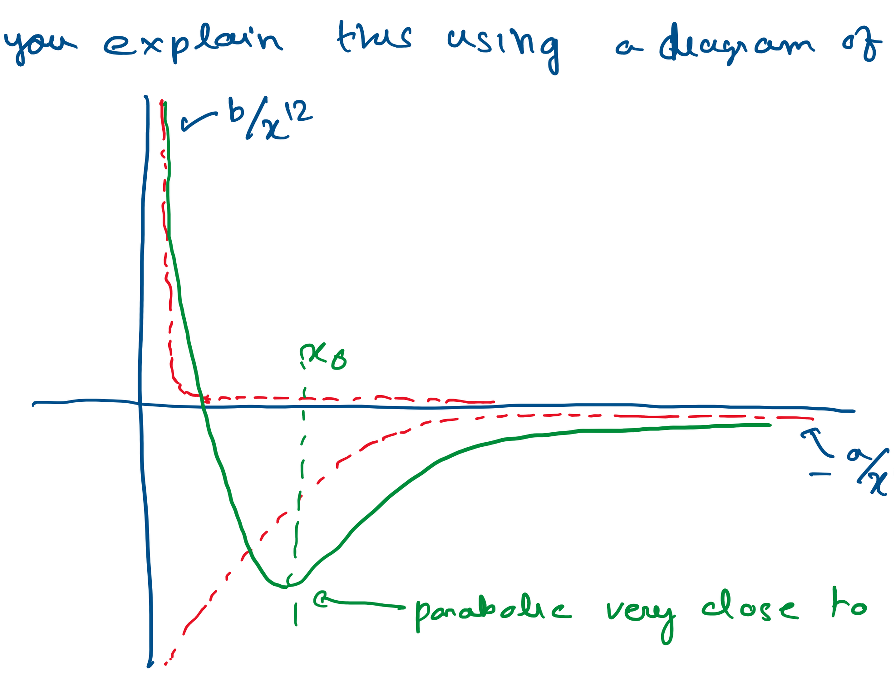
$$= \alpha\left[-\frac{6\Delta x}{x_0}\right]$$

$$= -\frac{36a}{x_0^8}(\Delta x)$$

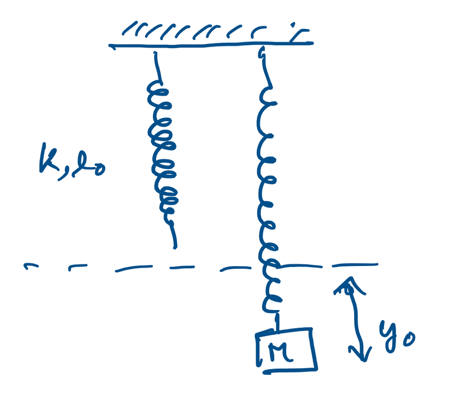
or $\underbrace{-\overbrace{\left(\frac{36a}{x_0^8}\right)}^{k}(x - x_0)}_{\text{SHO form by comparison}}$

$F = -kx$
or
$-k(x - x_0)$

iv> Can you explain this using a diagram of U(x)?



Q2> Lets solve for those springs that we have been comparing to



Strategy

i> Find the new eq point
ii> Perturb it slightly
iii> Find force when its slightly perturbed

At equilibrium

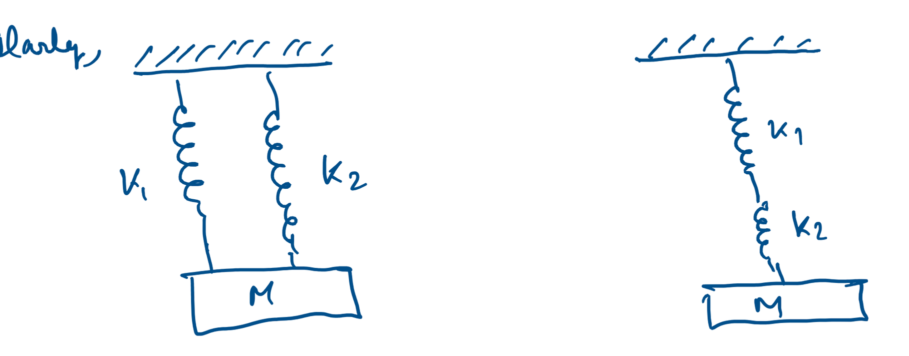
$F = 0 = mg - ky_0 \Rightarrow mg = ky_0$

With small perturbation

$$F = mg - k(y_0 + \Delta y)$$

$$m(\ddot{\Delta y}) = -k\Delta y \quad \leftarrow \text{use equilibrium conditions}$$

Similarly,



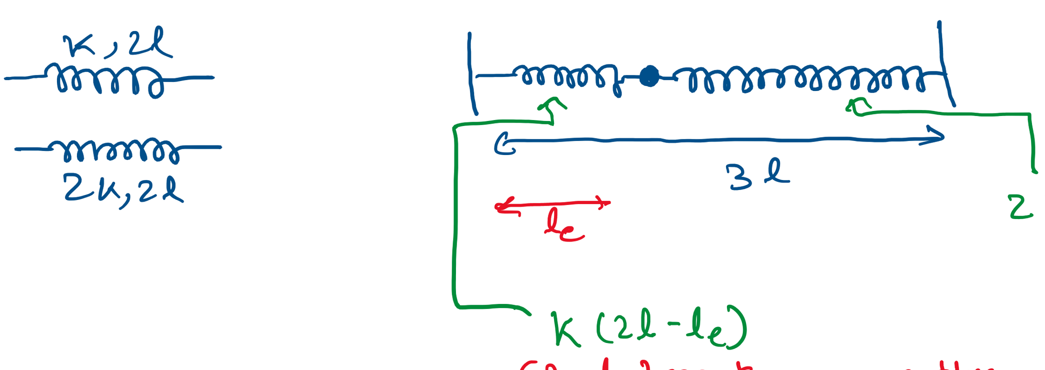
← May not solve these in class

$F = -(k_1 + k_2)\Delta y$

$F = -\frac{k_1 k_2}{k_1 + k_2}\Delta y$ } we use $k_1 x = k_2(\Delta y - x)$ replace

∴ $F = -k_2(\Delta y - x)$

Lets solve something trickier



$2k(2l - (3l - l_e))$

$k(2l - l_e)$

(feel free to assume this is compressed)

For eq,

$$k(2l - l_e) = 2k(2l - (3l - l_e))$$

$$l_e = 4l/3$$

∴ For small perturbation,

$$F = k(2l - (l_e + x)) - 2k(2l - (3l - l_e - x))$$

$$= -3kx$$

But what happens when forces are difficult. We take the energy approach

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \text{constant}$$

$$\therefore \frac{dE}{dt} = 0 = m\dot{x}\ddot{x} + kx\dot{x}$$

} This is a very useful approach as you go ahead

$$\Rightarrow m\ddot{x} = -kx$$

or

$$E = \frac{1}{2}mv^2 + U(x)$$

$$\frac{dE}{dt} = 0 = m\dot{x}\ddot{x} + \frac{dU}{dt}$$

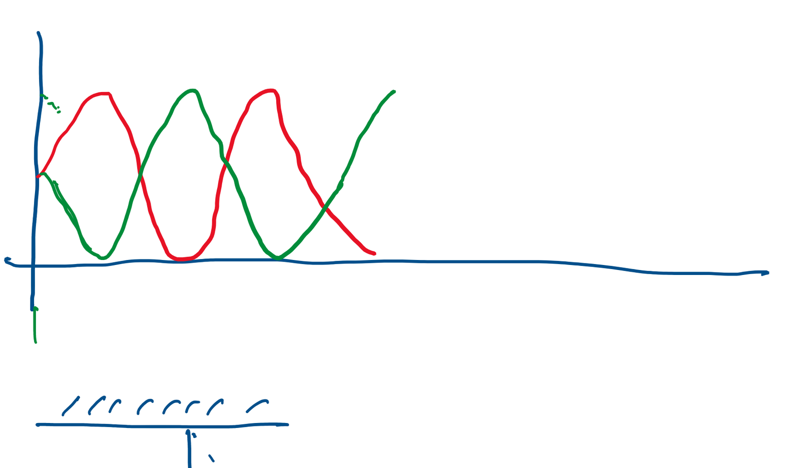
$$\Rightarrow 0 = m\dot{x}\ddot{x} + \underbrace{\frac{dU}{dx}\frac{dx}{dt}}_{\text{reminds you of something?}}$$

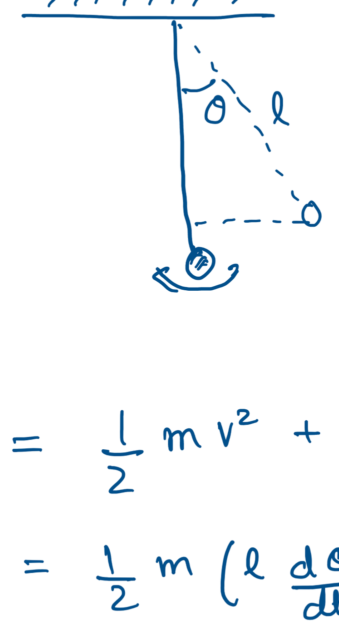
$$\Rightarrow m\ddot{x} = -\frac{dU}{dx}$$

So, for $x = A\cos(\omega t - \phi)$

$$K.E. = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\sin^2(\omega t - \phi)$$

$$P.E. = U(x) = \frac{1}{2}kx^2 = \frac{1}{2}kA^2\cos^2(\omega t - \phi)$$





Solve the pendulum SHO using the energy approach

$$E = \frac{1}{2}mv^2 + mgl(1 - \cos\theta)$$

$$\Rightarrow E = \frac{1}{2}m\left(l\frac{d\theta}{dt}\right)^2 + mgl(1 - \cos\theta)$$

$$\Rightarrow \frac{dE}{dt} = 0 = ml^2\dot{\theta}\ddot{\theta} + mgl\sin\theta\,\dot{\theta}$$

$$\Rightarrow \ddot{\theta} = -\frac{mgl}{ml^2}\sin\theta$$

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \dots$$

$$\Rightarrow \ddot{\theta} = -\frac{g}{l}\theta$$