Lecture 1

Tuesday, September 27, 2022 1:21 PM

Hey, I am Rajashik Tarafder. You can call me Ray.
And welcome to your favourite and most memorable course at Caltech.

Before, we jump into the gory details of physics, let me share some basic information:

TA's Information

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Rec Hours: 11-12 PM W/F, B111 DWNS
Office Hours: 7-8 PM Tuesdays, Zoom
Website: rtarafder.co.in/courses.html.
I will be posting all materials from the rec session here.

Like any course, I am heavily dependent on your feedback to get things right. So I will really appreciate it if you can drop me a line or two after classes at a feedback form on the website. I will love it even more when you let me know that you have enjoyed the class

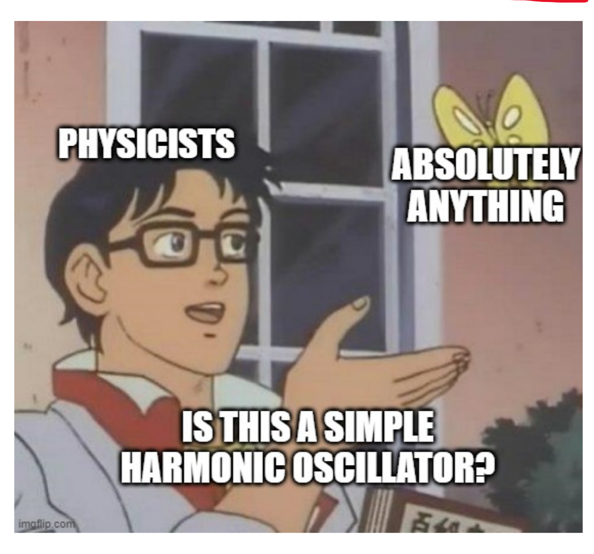
About me

I am a fifth year Graduate student and did my undergraduate from India. I used to do Astrophysics but am now working on Quantum Gravity. If you want to learn about these areas, feel free to ask me questions. No guarantees on my ability to answer them.

To learn a bit more about you, I shall pass some index cards and it would be awesome if you want to write down your names, pronouns, something interesting about you and something you wish to learn from me.

For the rec sessions I usually like to go with theory in the first 15-20 mins & then jump into problems. Today will be a bit different as it's the first class.

Why care about this course?



"The career of a young theoretical physicist is treating the HO in increasing levels of abstraction"

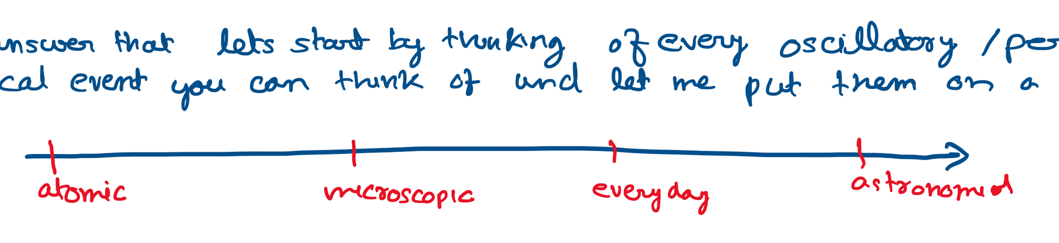
— Sydney Coleman (probably)

So instead of learning any new laws of nature in this course, we will learn on how to use our understanding of laws to solve the complex problems in physics.
Actually this is the basis of all engineering too.

We will just use Newton's laws in this course.

Now, you already know about these laws. So whats new?

To answer that lets start by thinking of every oscillatory / periodic physical event you can think of and let me put them on a scale

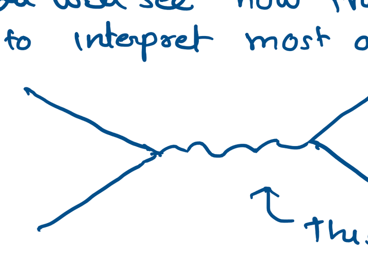


As we see, oscillations are everywhere.

So why is then Sergi ignoring all the fun stuff and talking to you about springs?
— in 400 years of physics, thats all we know how to solve (well!)

And we will see towards the end that this is more or less enough.

And in Ph2b you will see how the quantum mechanical version of this is enough to interpret most of the quantum world.



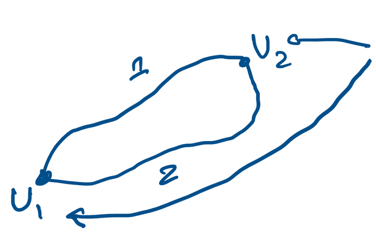
This is a harmonic oscillator.

And not just in physics, as you explore engineering, you will find yourself solving for "harmonics".

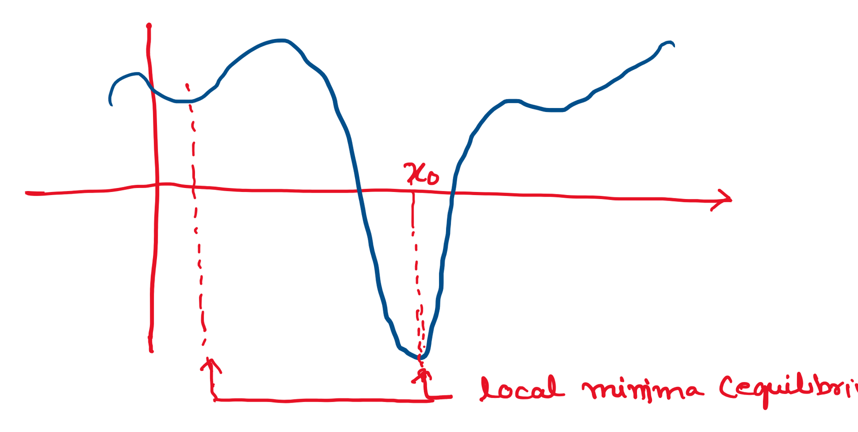
Conservative Forces

→ Forces for which the work done is independent of path

$$\oint \vec{F}\cdot d\vec{r} = 0$$



So lets take some arbitrary $U(x)$ (1-D) profile



As systems try to move into lowest energy states, much of physics can be solved by understanding what happens here.

So, using a ton of hindsight, let we write down $U(x)$ in this region as a taylor expansion.

$$U(x) = U(x_0) + \left.\frac{dU}{dx}\right|_{x_0}(x-x_0) + \frac{1}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2 + \cdots$$

[Because $U_2 - U_1 = \int_1^2 \vec{F}\cdot d\vec{r}$
$\vec{F} = -\frac{dU}{dr}\hat{r}$]

In our case $F = -\frac{dU}{dx}$ | more generally $F_x = -\frac{\partial U}{\partial x}$

$$\therefore F(x) = 0 - \left.\frac{dU}{dx}\right|_{x_0} - \left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0) - \frac{1}{2!}\left.\frac{d^3U}{dx^3}\right|_{x_0}(x-x_0)^2 \cdots\cdots$$

Now, if we use the fact that at local minima $\left.\frac{dU}{dx}\right|_{x_0} = 0$

no approximations so far

near x_0

$$F(x) = -\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0) - \cdots\cdots \approx -\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)$$

near x_0

$$U(x) = U(x_0) + \frac{1}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2 + \cdots \approx U(x_0) + \frac{1}{2!}\left.\frac{d^2U}{dx^2}\right|_{x_0}(x-x_0)^2$$

harmonic terms

The next harmonic term gives you Feynman diagrams.

Now, if you remember, we argued in Ph1 that physics doesnt depend on reference frames. Also, $U(x_0)$ is a arbitrary constant term (invariances)

$$F(x) = -\frac{d^2U}{dx^2}x$$

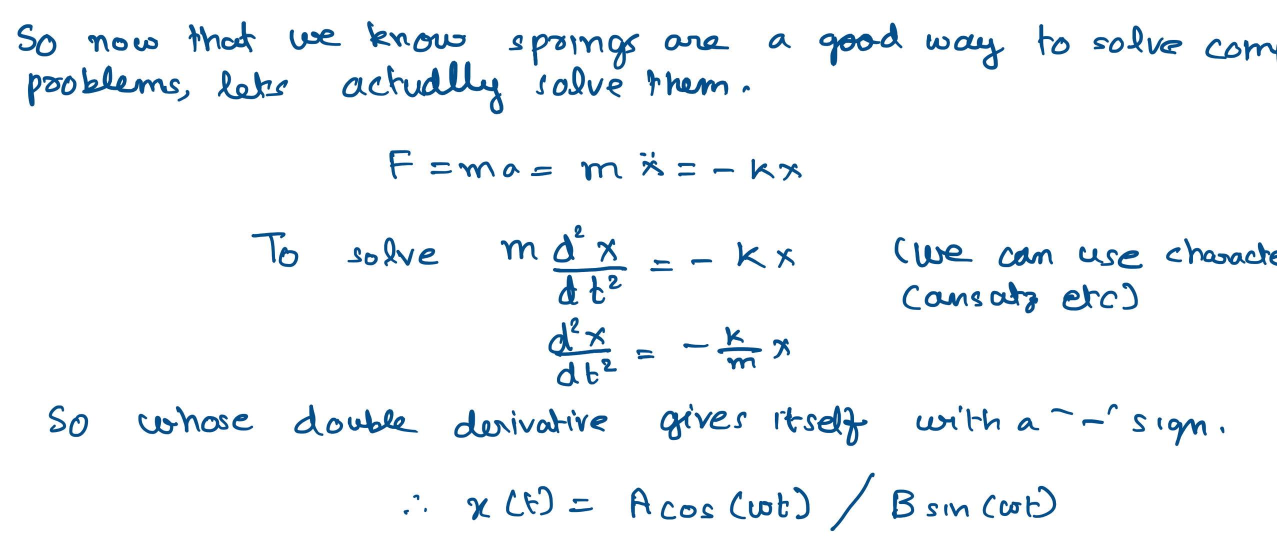
$$U(x) = \frac{1}{2}\frac{d^2U}{dx^2}x^2$$

Setting $\frac{d^2U}{dx^2} = k$

spring constant

$$\boxed{\begin{aligned} F(x) &= -kx \\ U(x) &= \tfrac{1}{2}kx^2 \end{aligned}} \longrightarrow \text{SHO equations}$$

So we just simplified the most generalized conservative forces to a set of equations that describe springs



So now that we know springs are a good way to solve complex problems, lets actually solve them.

$$F = ma = m\ddot{x} = -kx$$

To solve $m\frac{d^2x}{dt^2} = -kx$ (we can use characteristic eqs) (ansatz etc)

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x$$

So whose double derivative gives itself with a '−' sign.

$$\therefore x(t) = A\cos(\omega t) \,/\, B\sin(\omega t)$$

plugging it in we see that $\omega = \sqrt{k/m}$

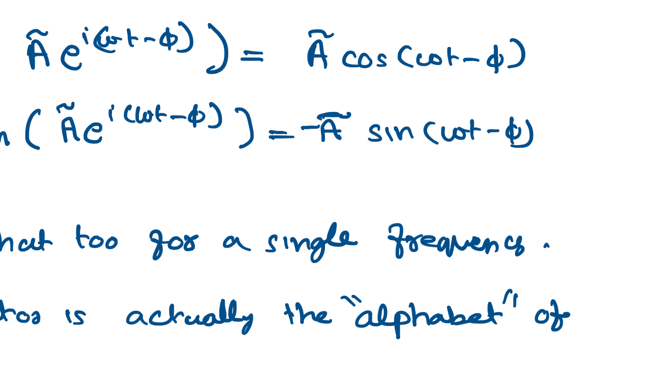
$$\therefore x(t) = A\cos(\omega t) + B\sin(\omega t)$$

freq

or

$$\tilde{A}\cos(\omega t - \phi)$$

$f = \omega/2\pi$
$T = 2\pi/\omega$



Thats expected as every 2nd degree ODE needs two constants to be solved.

Thats all you need.

We can use $x(0)$ & $\dot{x}(0)$ to find these out.

$A = x(0)$ | $\tilde{A} = \sqrt{x(0)^2 + \dot{x}(0)^2}$

$B = \dot{x}(0)$ | $\phi = \tan^{-1}\left(-\frac{\dot{x}(0)}{x(0)}\right)$

But lets also look at its complex form as its far more useful.

$$x(t) \longrightarrow z(t)$$

$$\ddot{z} = -\omega^2 z$$

$$\therefore z(t) = \tilde{A}e^{i(\omega t - \phi)} \,/\, Ae^{i\omega t} + Be^{-i\omega t}$$

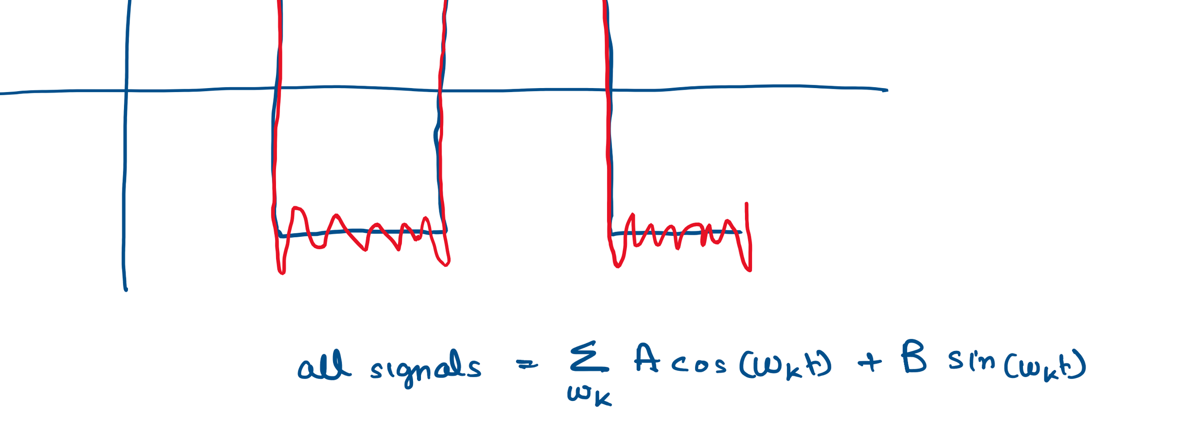
Its quick to check that,

$$x(t) = \mathrm{Re}\left(\tilde{A}e^{i(\omega t - \phi)}\right) = \tilde{A}\cos(\omega t - \phi)$$

$$\dot{x}(t) = -\mathrm{Im}\left(\tilde{A}e^{i(\omega t - \phi)}\right) = -\tilde{A}\sin(\omega t - \phi)$$

But all this effort for just springs that too for a single frequency.

A single frequency harmonic oscillator is actually the "alphabet" of all periodic motion.



$$\text{all signals} = \sum_{\omega_k} A\cos(\omega_k t) + B\sin(\omega_k t)$$

This process known as fourier decomposition is central to how the world works.