Formula Sheet 1

Thursday, October 13, 2022 4:19 PM

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos(\omega t)$$

Driving frequency

Natural frequency

viscous damping

① Simple Haromonic Oscillator (only two terms)

$$\ddot{x} + \omega_0^2 x = 0$$

Soln. $\longrightarrow$ $x(t) = A \cos(\omega t + \phi)$

or

$$z(t) = \tilde{A} e^{i(\omega t + \phi)}$$

define from initial conditions

Useful relations

⊛ $\omega_0 = \sqrt{k/m}$ for spring-mass system

② Damped Harmonic Oscillator (if a damping term is added)

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = 0$$

Three classes of solution:

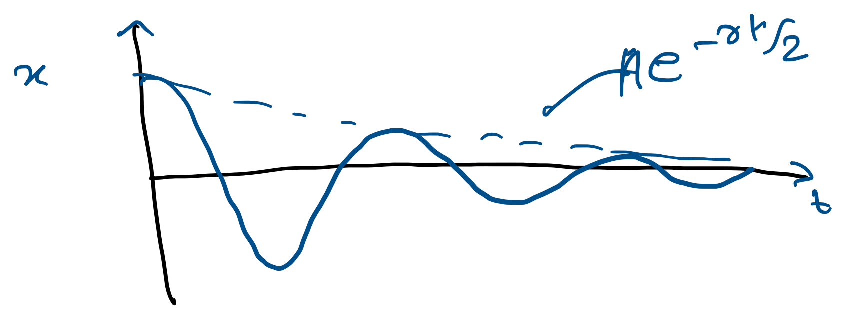
(i) $\gamma < 2\omega_0$ (underdamped)

$$x(t) = A e^{-\gamma t/2} \cos(\omega_u t + \phi)$$

determined from initial condition

$$\omega_u = \omega_0 \sqrt{1 - \frac{\gamma^2}{4\omega_0^2}}$$

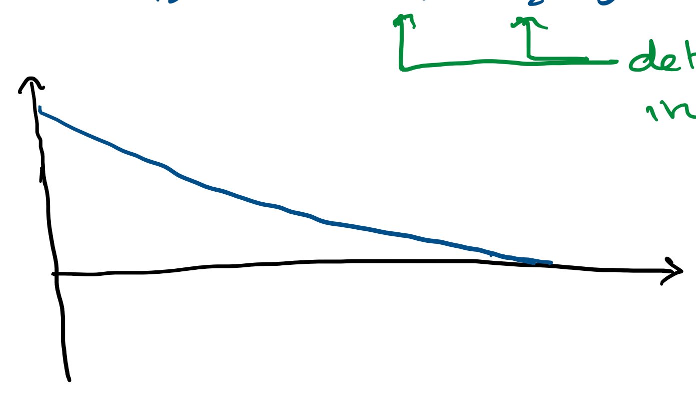
(oscillations)



(ii) $\gamma = 2\omega_0$ (critical damping)

$$x(t) = (C_1 + C_2 t) e^{-\gamma t/2}$$

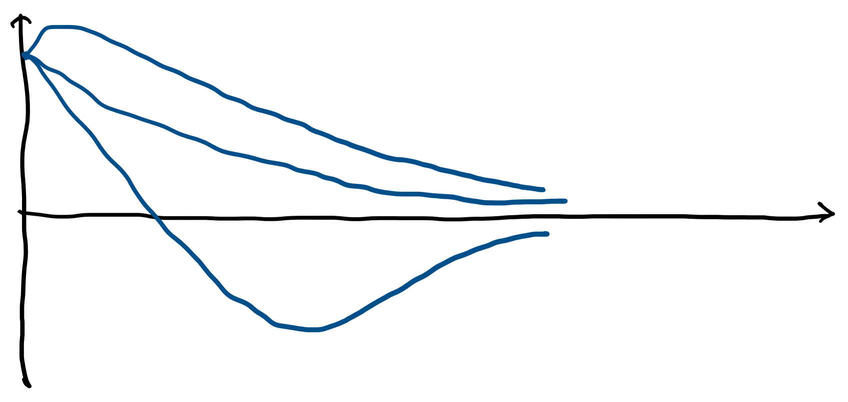
determined from initial condition



(iii) $\gamma > 2\omega_0$ (overdamping)

$$x(t) = C_1 e^{-\alpha_1 t} + C_2 e^{-\alpha_2 t}$$

where $\alpha_{1,2} = \frac{\gamma}{2} \pm \sqrt{\frac{\gamma^2}{4} - \omega_0^2}$



Energy loss $= -b\dot{x}^2 = -\gamma m \dot{x}^2$

③ Driven Harmonic Oscillators

$$\ddot{x} + \gamma \dot{x} + \omega_0^2 x = \frac{F_0}{m} \cos(\omega t)$$

Only 1 soln again.

$$x(t) = \underbrace{x_{damped}}_{\text{transient (dies out)}} + \underbrace{A \cos(\omega t - \delta)}_{\text{steady state}}$$

$$A(\omega) = \frac{F_0/m}{[(\omega_0^2 - \omega^2)^2 + \gamma^2 \omega^2]^{1/2}}$$

$$\delta(\omega) = \tan^{-1}\left(\frac{\gamma \omega}{(\omega_0^2 - \omega^2)}\right)$$