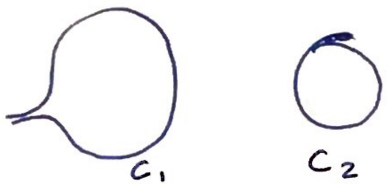
Lecture 8

Due to current I in a wire element $\vec{B}$ anywhere in space is $\propto I$.

$\therefore \phi \propto I$.

$\therefore$ for two wire loops



$$\phi_{21} = (\quad) I_1$$

(flux in 2 due to current in 1.

The proportionality factor is called mutual inductance (M)

Properties

1> It is a property of the geometry of two loops.

2> $M_{21} = M_{12}$

When we calculate the inductance of the loop on itself, we call it self inductance (L or M_{11})

We refer to it as inductance as it is the proportionality constant for induced $\mathcal{E}$ due to change in current.

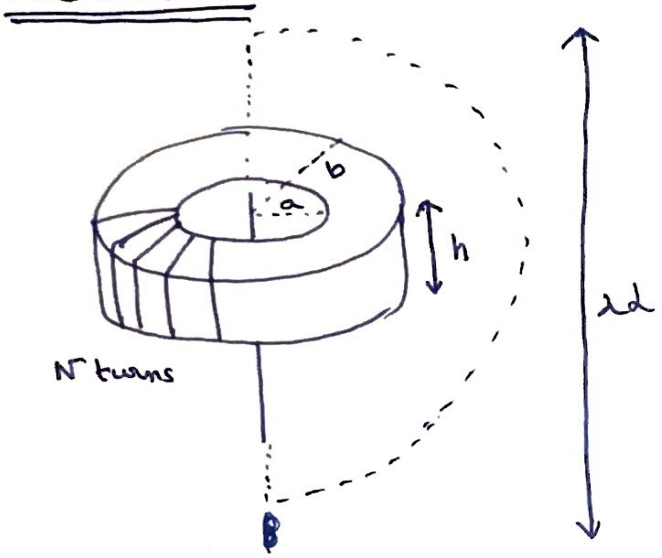
$$\mathcal{E}_{21} = - M_{21}\, dI_1/dt .$$

Unit of Inductance is $\frac{\text{volt} \times \text{sec}}{\text{amp}}$ or Henry.

⊛ Self and mutual inductance can only be defined under quasi-static approximation.

Let us look at a few problems right away. They are intended to help ~~solve~~ study a few techniques.

Problem 1



A toroidal coil with rectangular cross section has N evenly spaced tightly packed turns.

Inner radius – a

Outer radius – b

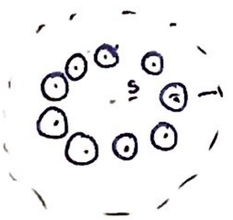
Height – h

A separate long straight wire is placed along the axis of the toroid extending from $z=-d$ to $z=d$. The ends are connected by a semicircle of radius d forming a wire loop.

a) Find Mutual inductance. ($M_{\text{toroid due to loop}}$)

b) The axial wire is now displaced by $d<a$ towards one side of the torus. What is the new mutual inductance?

a) We choose to find the flux through the wire loop due to current in the toroid.



$$B(s) = \frac{\mu_0 N I}{2\pi s}$$

$$\therefore \Phi_{\text{through wire loop}} = \int \vec{B} \cdot d\vec{a}$$

$$= \frac{\mu_0 N I}{2\pi} \int_a^b \frac{h\,ds}{s}$$

$$= \frac{\mu_0 N I h}{2\pi} \ln\left(b/a\right)$$

$$\therefore M = \frac{\mu_0 N h}{2\pi} \ln\left(b/a\right)$$

b) The flux remains the same. $\therefore$ M is same.

Problem 2

A short solenoid of length 'l' and radius 'a' with n_1 turns per unit length lies on the axis of a very long solenoid of radius b and n_2 turns per unit length. Current I flows through the short solenoid. What is the flux through the long solenoid?

As the inner solenoid is short, it's field is ~~...~~ very complicated.

∴ We choose an alternate method of finding M_{sl} and then using it to find the flux in the long solenoid.

$$B_l = \mu_0 n_2 I$$

$$\phi_{sl} = \mu_0 n_2 I \; \pi a^2 n_1 l$$

$$\therefore M_{sl} = \mu_0 n_1 n_2 \pi a^2 l$$

$$\therefore \phi_{ls} = M_{ls} I$$

$$= M_{sl} I$$

$$= \mu_0 n_1 n_2 \pi a^2 l I$$

Problem 3

Find the magnetic field strength due to a ring current at points in the plane of the ring much greater than the radius.

Let us choose an outer ring at radius R_1 with current I_1

∴ B_1 at the center due to outer ring $= \mu_0 I_1 / 2R_1$

$$\therefore \phi_{21} = \pi R_2^2 B_1 = \frac{\mu_0 \pi R_2^2}{2R_1} I_1$$

If we slightly change the outer radii $\Delta\phi_{21} = \frac{\partial \phi_{21}}{\partial R_1} \Delta R_1 = -\frac{\mu_0 \pi R_2^2}{2R_1^2} I_1 \Delta R$

We try the same process with a current I_2 in the inner ring.

$$\cancel{\phi} \quad \Delta \Phi_{12} = - B_2 \, 2\pi R_1 \Delta R_1$$

As the mutual inductance is equal $(M_{12} = M_{21})$

$$\frac{\Delta \Phi_{12}}{I_2} = \frac{\Delta \Phi_{21}}{I_1}$$

$$\Rightarrow B_2 = \frac{\mu_0 R_2^2 I_2}{4 R_1^3}$$

Problem 4

Compute the self inductance of the setup shown below

$$B_1 = \frac{\mu_0 I}{2\pi s} \quad \text{(due to 1 arm)}$$

$$\Phi_{total} = 2 \frac{\mu_0 I}{2\pi} l \int_{\epsilon}^{d-\epsilon} \frac{ds}{s} = \frac{\mu_0 I l}{\pi} \ln\left(d-\epsilon/\epsilon\right) = \frac{\mu_0 I l}{\pi} \ln\left(d/\epsilon\right)$$

The choice of ϵ is an essential tool in several self inductance problems. ~~Infinitely~~ Absolutely thin wires tend to blow up solutions