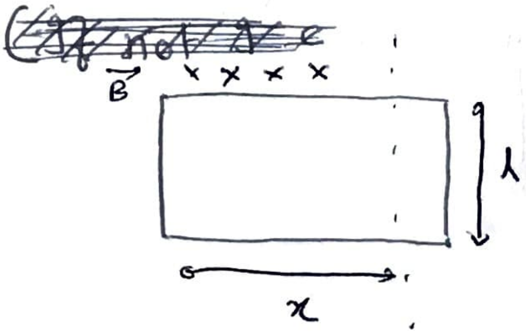
Lecture 6

For today's class, we will be very quick with derivations and focus on problems to understand Faraday's law.

Faraday & Lenz's law together tell us that systems like to avoid change in the magnetic flux. This is done by developing which sends charges around a loop and creating a magnetic field.

This is usually written as $\mathcal{E} = -\frac{d\phi}{dt}$ where ϕ is the flux through a loop of any shape in a magnetic field.

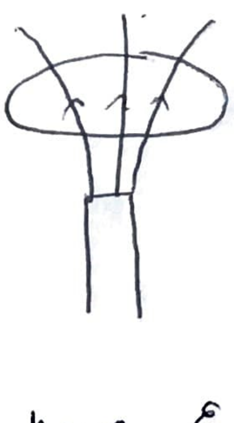


$$\phi = Blx$$

$$\mathcal{E} = -\frac{d\phi}{dt} = Blv$$

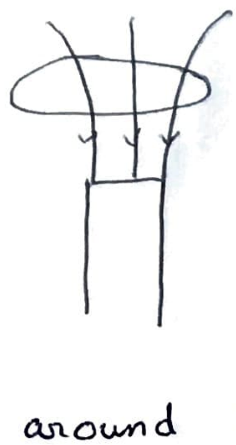
$$\mathcal{E} = -(B_1 - B_2)lv \quad \text{(general form)}$$

Always use Lenz law to determine direction.



i> ↑ v

ii> ↓ v



i> ↑ v

ii> ↓ v

As we have $\mathcal{E}$ when we go around a loop, the field cannot be electrostatic.

i.e. $\vec{\nabla} \times \vec{E} \neq 0$. $\mathcal{E} = \int \vec{E} \cdot d\vec{l} = \int \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\int \frac{d\vec{B}}{dt} \cdot d\vec{a}$

$$\therefore \vec{\nabla} \times \vec{E} = -\partial \vec{B} / \partial t$$

$\mathcal{E}$ can be understood as the energy per unit charge that the system imparts to a carrier particle as it goes around a loop.

In the case of surfaces, where there is no loop, we end with eddy currents.

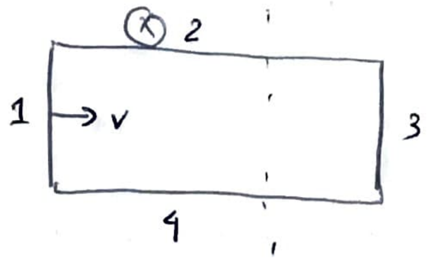
∴ motion feels sluggish as if moving through a viscous fluid.

However, if we can somehow restrict large scale currents, we can reduce sluggishness

— slits prevent large currents.

We shall also look at a different form of this $\mathcal{E}$ which is given as

$$\mathcal{E} = \int \vec{f} \cdot d\vec{l}.$$



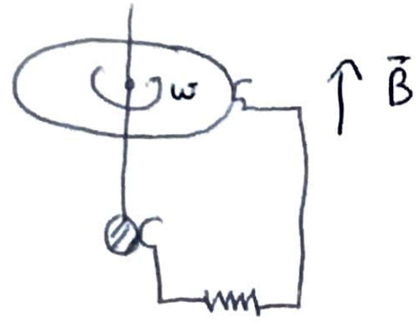
$$\mathcal{E} = \int \vec{f} \cdot dl + \int_2 + \int_3 + \int_4$$

$$= \int_0^l (\vec{v} \times \vec{B}) \cdot dl + 0 + 0 + 0$$

$$= vBl$$

All the $\mathcal{E}$ is along vertical section alone.

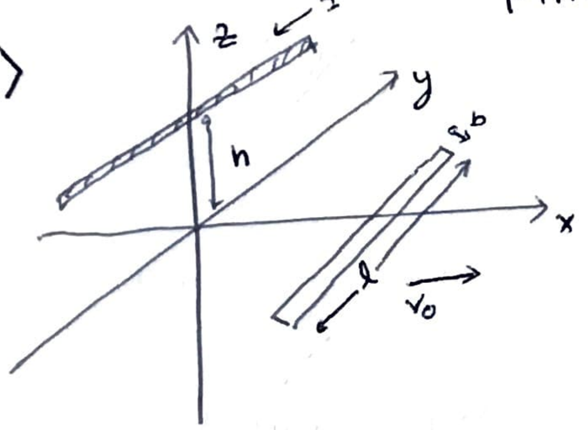
1>



A metal disk of radius 'a' rotates with angular velocity 'ω' about a vertical axis through a uniform $\vec{B}$ pointing upwards. A circuit is made by connecting one end of the resistor to the axle and the other end to ~~a sliding~~ the edge of the disk.

Find current R?

2>



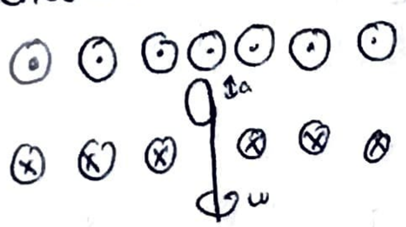
A long straight wire is parallel to the y axis and passes through z = h. A current flows through it in the $-\hat{y}$ direction.

A thin rectangular loop lies on the x axis with length 'l' parallel to the wire and small width 'b'. The loop slides with constant V_0 in the $\hat{x}$ direction.

Find $\mathcal{E}$ and at what point is ~~maximum~~ it maximum?

Hint: Use center of loop as its location

3> Mechanical DC to AC Convertor



A DC current is sustained by V_0 voltage across a solenoid of Resistance R_s with n turns per unit length.

A circular loop of wire with radius 'a' and resistance R_l is placed in the solenoid ⊥ to the magnetic field. The loop is allowed to rotate about an axis ⊥ to the axis of the solenoid. Find

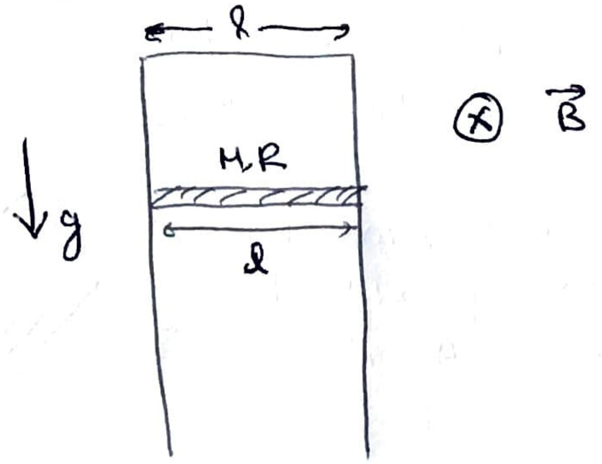
a> Magnetic field of solenoid

b> Φ as a function of time

c> $\mathcal{E}$ as a function of time & I as a function of time in the loop.

d> As $\vec{B}$ remains constant, where is the energy ~~[illegible]~~ obtained for current in ~~[illegible]~~ loop.

4) A bar of Mass M, resistance R and length l slides on a vertical frictionless conducting rails. The rails are connected by a wire at the top. The resistance of rails and wire is negligible.



$\vec{B}$ points into the paper and uniform g acts downward. At $t=0$, the bar is released with $v=0$. (Ignore $\vec{B}$ due to induced current).

a) Calculate $\mathcal{E}$ & I in the loop

b) Find the force experienced by the bar

c) Find terminal velocity of bar

d) Compute rate of gravitational energy loss when bar reaches terminal velocity. How is the energy lost.

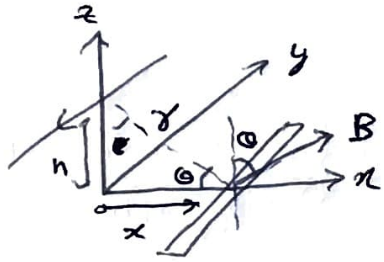
1) f_{mag} at any point of the disc

$$= \vec{v} \times \vec{B} = \omega r B \hat{r}$$

$$\therefore \mathcal{E} = \int_0^a \vec{f_{mag}} \cdot \vec{dr} = \omega B \int_0^a r\,dr = \frac{\omega B a^2}{2}$$

$$I = \mathcal{E}/R = \omega B a^2/2R$$

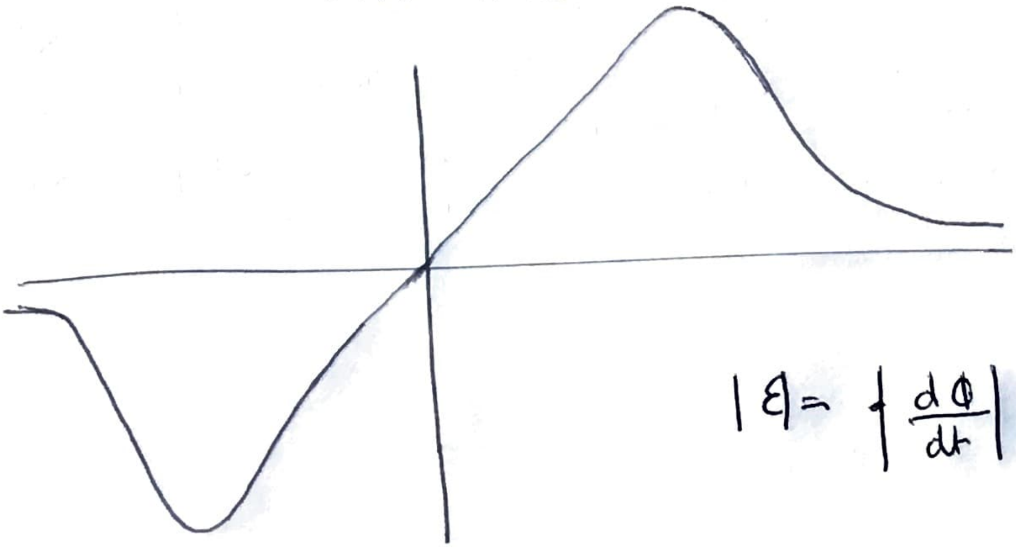
2) As $\phi = \int \vec{B} \cdot d\vec{a}$ and loop lies in the xy plane, we are only concerned with B_z



$$B_z(x) = \frac{\mu_0 I}{2\pi r} \cos\theta$$

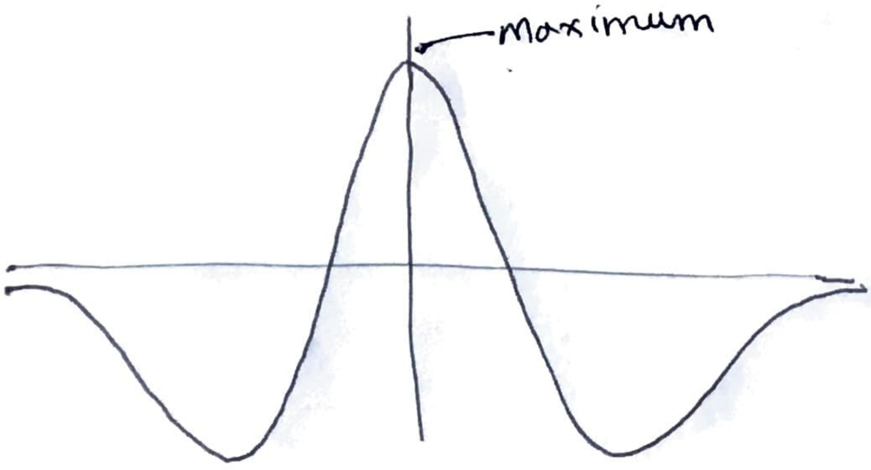
$$= \frac{\mu_0 I}{2\pi r} \frac{x}{r}$$

$$= \frac{\mu_0 I}{2\pi r} \frac{x}{h^2 + x^2}$$



$$B_z \propto \frac{x}{h^2 + x^2}$$

$$|\mathcal{E}| = \left|\frac{d\phi}{dt}\right| = \frac{dB_z}{dt} A = \frac{dB_z}{dx} \frac{dx}{dt} A$$



$$= \frac{\mu_0 I}{2\pi} \left(\frac{1}{h^2 + x^2} + \frac{x(-2x)}{(h^2 + x^2)^2} \right) V_0 l b$$

$$= \frac{\mu_0 I}{2\pi} \frac{h^2 - x^2}{(h^2 + x^2)^2} V_0 l b$$

Local extremum at

$$d\mathcal{E}/dx = 0 \Rightarrow (h^2+x^2)^2(-2x) - (h^2-x^2)2(h^2+x^2)(2x) = 0$$

$$\Rightarrow x = 0 \quad \text{or} \quad x = \pm\sqrt{3}\,h$$

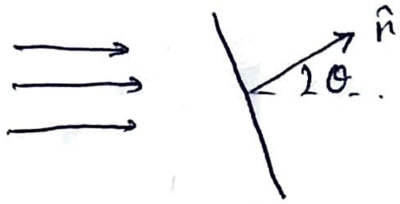
Second derivatives tell us where the maximum & minma are

3)

a) $I = V_0/R_s$

$$B = \mu_0 n I = \mu_0 n V_0/R_s$$

b)



$$\phi = \int \vec{B}\cdot d\vec{a}$$
$$= B\pi a^2 \cos\theta$$
$$= B\pi a^2 \cos(\omega t)$$

c) $$\mathcal{E} = -\frac{d\phi}{dt} = \frac{\mu_0 \pi a^2 \omega n V_0}{R_s} \sin(\omega t)$$

$$I_l = \frac{\mu_0 \pi a^2 \omega n V_0}{R_l R_s} \sin\theta$$

d) Energy is requred to turn the loop.

a) $\Phi_B = \int \vec{B} \cdot d\vec{a} = Blh$ ← arbitrary distance from the top

$$|\mathcal{E}| = \left|\frac{d\Phi}{dt}\right| = Blv$$

$$I = \mathcal{E}/R = \frac{Blv}{R} \quad \text{(anticlockwise)}$$

b) $\vec{F}_{mag} = BIl\,\hat{y} \quad (I\, l\hat{x} \times B(-\hat{z})) = \frac{B^2 l^2 v}{R}$

c) For terminal velocity

$$\vec{F}_{mag} + \vec{F}_g = 0$$

$$\Rightarrow BIl - Mg = 0$$

$$\Rightarrow \frac{B^2 l^2 v_{term}}{R} - Mg = 0$$

$$\Rightarrow v_{term} = \frac{MgR}{B^2 l^2}$$

d) $U_g = Mgy$

$$\frac{dU_g}{dt} = -Mg\,v(t)$$

$$\left.\frac{dU_g}{dt}\right|_{v_{term}} = -Mg\left(\frac{MgR}{B^2 l^2}\right) = -R\left(\frac{Mg}{Bl}\right)^2$$

This is lost as heat in the bar due to resistance.