Lecture 5

When deriving magnetism from first principles, it is sufficient to show $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{\theta}$ for a current carrying wire in the $\hat{z}$ direction. Dave has already done this in class. We will take a slightly different route to discuss a bit of physics along the way.

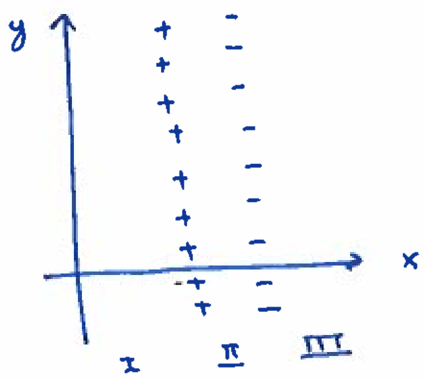
To derive magnetism, you usually need four postulates.

1> The laws of physics are same in all inertial reference frames
2> The speed of light is "c" and same in all inertial frames. } SR

We just add two more to the list.

3> Charge is invariant in all reference frame
4> Coulomb's law holds $\longrightarrow$ already holds due to 1.

Now, consider two infinite sheets of charge at rest in F frame as shown below. Both sheets have σ charge density.



In I, $\vec{E} = 0$

In II, $\vec{E} = \frac{\sigma}{\epsilon_0}\hat{x}$

In III, $\vec{E} = 0$

Now consider a frame F' moving with a velocity v to the left $\therefore$ in F' frame

σ & $-\sigma$ remain as charge density as lorentz contraction is only in the direction of motion.

$\therefore$ In I, $\vec{E}' = 0$

In II, $\vec{E}' = \frac{\sigma}{\epsilon_0}\hat{x}$

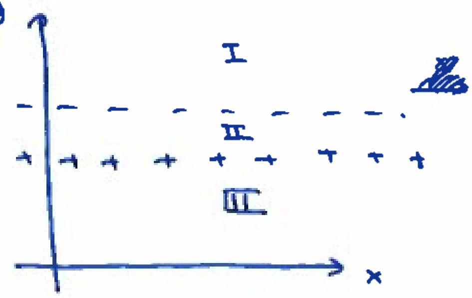
In III, $\vec{E}' = 0$

We call this $E_{\parallel}$ ~~and~~ as it is in the direction of motion.

$$\therefore \vec{E}'_{\parallel} = \vec{E}_{\parallel}$$

Applying the same logic to sheets in the xz plane.

In $\vec{F}$,

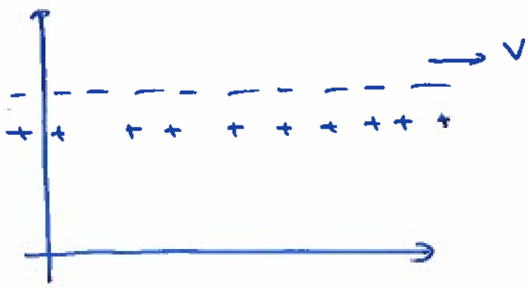


In I, $\vec{E} = 0$

In II, $\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{y}$

In III, $\vec{E} = 0$

In $\vec{F}'$, there is length contraction in the direction of motion.



$\therefore \sigma' = \sigma\gamma$

$\therefore$ In I, $\vec{E}' = 0$

In II, $\vec{E}' = \frac{\sigma\gamma}{\varepsilon_0}\hat{y}$

In III, $\vec{E} = 0$

We call this $E_{\perp}$ as it is $\perp$ to the direction of motion.

$$\therefore E'_{\perp} = \gamma E_{\perp}$$

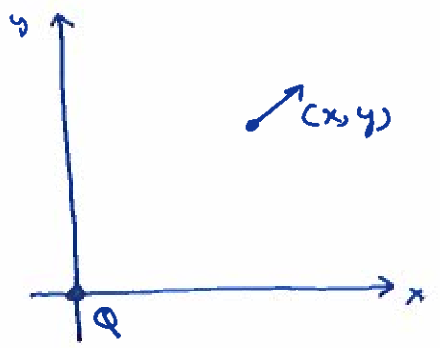
An immediate consequence of this,

$$\vec{F}'_{\parallel} = q E'_{\parallel} = q E_{\parallel} = F_{\parallel} \Rightarrow F'_{\parallel} = F_{\parallel}$$

$$F'_{\perp} = q E'_{\perp} = q\gamma E_{\perp} = \gamma F_{\perp} \Rightarrow F'_{\perp} = \gamma F_{\perp}$$

Lets take a few minutes off and see how the field looks like.

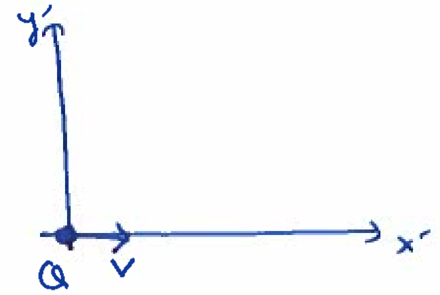
For a point charge at the origin in F frame, we calculate the $\vec{E}$ in a general plane. Here, we shall use xy plane for convenience.



$$E_x = \frac{Qx}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}$$

$$E_y = \frac{Qy}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}$$

In F′ frame which coincides with F frame at the origin at $t' = t = 0$



At $t = 0$

$$E_x' = E_x = \frac{Qx}{4\pi\varepsilon_0 (x^2+y^2)^{3/2}}$$

when expressed in x' & y'

$$= \frac{Q(\gamma x')}{4\pi\varepsilon_0 \left[(\gamma x')^2 + y'^2\right]^{3/2}}$$

$$\& \; E_y' = \gamma E_y$$

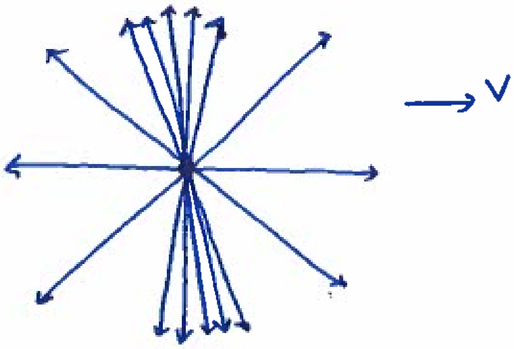
$$= \gamma \frac{Q(y')}{4\pi\varepsilon_0 \left[(\gamma x')^2 + (y')^2\right]^{3/2}}$$

We skip some mathematics here,

$$E' = \sqrt{E_x'^2 + E_y'^2} = \frac{Q}{4\pi\varepsilon_0 (x'^2+y'^2)} \frac{(1-\beta)^2}{(1-\beta^2\sin^2\theta')^{3/2}}$$

where $\theta' = \sin^{-1}\left(\frac{y'}{(x'^2+y'^2)^{1/2}}\right)$

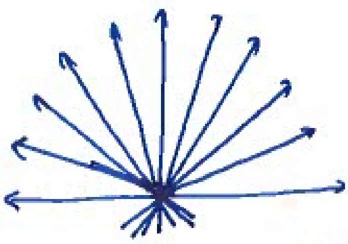
On plotting this,



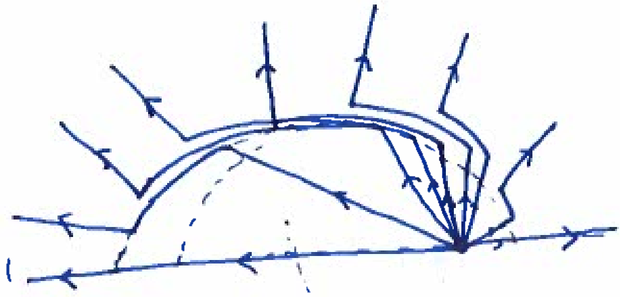
This field has non zero curl. You cannot define a vector potential for it.

Plots for charges accelerating ~~or deceeler~~ in jerks. (explanation elaborated in class)

$t = t_i, \ v = 0$

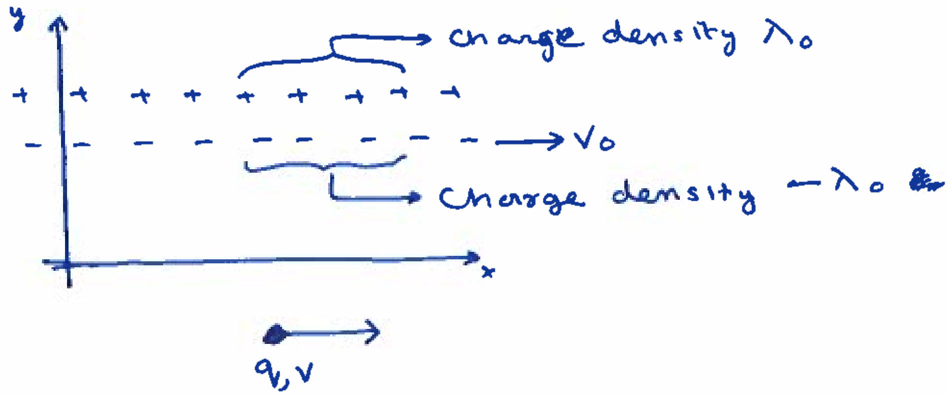


$t = t_f, \ v = v_f$



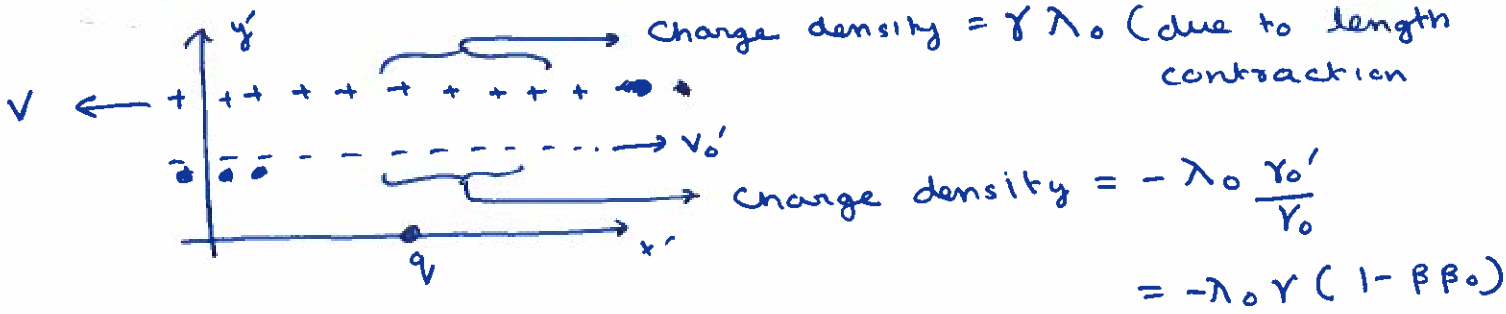
Bad drawing skills. Refer to Purcell. Fig 5.18.

Now, as derived in the class,



As there is no net charge density, there is no $\vec{E}$ due to wire in this frame.

In the frame of moving charge,



Here $v_0' = \dfrac{v_0 - V}{1 - Vv_0/c^2}$

$\therefore$ Total charge density $= \gamma \lambda_0 \beta \beta_0$

$$\therefore E_y' = \frac{\gamma \beta\beta_0 \lambda_0}{2\pi\epsilon_0 r'} (-\hat{y})$$

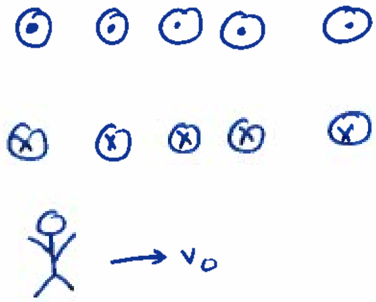
$$\Rightarrow F_y = \frac{F_y'}{\gamma} = \frac{qE_y'}{\gamma} = \frac{q\beta\beta_0\lambda_0}{2\pi\epsilon_0 r}(-\hat{y})$$

Now $-\lambda_0 v_0 = I$

$$\therefore F_y = q v_x \frac{I}{2\pi\epsilon_0 r c^2} = q v_x \frac{\mu_0 I}{2\pi r}$$

Relativistic Solenoid

Conside a infinitely long solenoid carrying current I. Let 'n' deno the number of turns of wire per unit length. An observor travels along the solenoid with the velocity v_0



a) What is the $|\vec{B}|$ in the solenoid's rest frame.

$$\vec{B} = \mu_0 n I$$

b) What is the current in the observor's frame? What is the number density for turns?

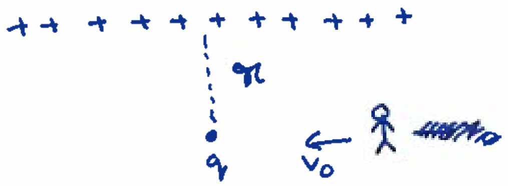
$$I' = \frac{dq}{dt'} = \frac{dq}{\gamma dt} = \frac{I}{\gamma}$$

$$n' = \frac{N}{L'} = \frac{N\gamma}{L} = \gamma n$$

c) What is the magnetic field in the frame of the observor?

$$\vec{B}' = \mu_0 n' I' = \mu_0 n I = B$$

Relativistic Wire



We neglect E & B by the point charge.

a) What is $\vec{E}$ & $\vec{B}$ by the wire in the frame of the charge? Find the total force.

$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}(-\hat{y})$$

$$\vec{F} = \frac{\lambda q}{2\pi\varepsilon_0 r}(-\hat{y})$$

b) What is the charge density in the frame of the observer? What is the current.

$$\lambda' = \gamma\lambda$$

$$\therefore I' = \lambda' v_0 = \gamma\lambda v_0\,(\hat{x})$$

c) What is $\vec{E}$ & $\vec{B}$ in the observor's frame

$$\vec{E} = \frac{\gamma\lambda}{2\pi\varepsilon_0 r}(-\hat{y})$$

$$\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{z}) = \frac{\mu_0\gamma\lambda v_0}{2\pi r}(-\hat{z})$$

$$F_{net} = q\left(\frac{\gamma\lambda}{2\pi\varepsilon_0 r}(-\hat{y}) + v_0\hat{x} \times \frac{\mu_0\gamma\lambda v_0}{2\pi r}(-\hat{z})\right)$$

$$= q\left(\frac{\gamma\lambda}{2\pi\varepsilon_0 r}(-\hat{y}) + \frac{\mu_0\gamma\lambda v_0^2}{2\pi r}(\hat{y})\right) = \gamma\frac{q\lambda}{2\pi\varepsilon_0 r}\left[1 - \mu_0\varepsilon_0 v^2\right]$$

$$= \frac{1}{\gamma}\frac{q\lambda}{2\pi\varepsilon_0 r}(-\hat{y})$$