Formula Sheet - 3

Impedance

$$Z_R = R$$
$$Z_L = i\omega L$$
$$Z_C = \frac{1}{i\omega C}$$

You can treat impedances the way you find equivalent resistances and apply Kirchoff's laws.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Power in AC circuits: $V_{rms} I_{rms} \cos\phi$

Phase lag of current & applied voltage.

Maxwell's Laws

$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0 \quad , \quad \vec{\nabla}\cdot\vec{B} = 0$$
$$\vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} \quad , \quad \vec{\nabla}\times\vec{B} = \mu_0\vec{J} + \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

Integral form

$$\oint_S \vec{E}\cdot d\vec{a} = \int_V \frac{\rho\, dv}{\varepsilon_0} \quad , \quad \oint_S \vec{B}\cdot d\vec{a} = 0$$

$$\oint_C \vec{E}\cdot d\vec{l} = -\int_S \frac{\partial\vec{B}}{\partial t}\cdot d\vec{a} = -\frac{\partial}{\partial t}\int_S \vec{B}\cdot d\vec{a} = -\frac{\partial\Phi_B}{\partial t}$$

$$\oint_C \vec{B}\cdot d\vec{l} = \int \mu_0\vec{J}\cdot d\vec{a} + \int \mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}\cdot d\vec{a} = \mu_0 I_{enc} + \mu_0\varepsilon_0\frac{\partial\Phi_E}{\partial t}$$

Equation of forward travelling wave is $f(x-vt)$

Equation of backward travelling wave is $f(x+vt)$

speed : v

Differential form of travelling wave : $\frac{\partial^2 f}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2 f}{\partial t^2}$

General properties of EM Wave

1) travel with speed 'c' = $\frac{1}{\sqrt{\mu_0 \varepsilon_0}}$

2) at every point $|\vec{E}| = c\,|\vec{B}|$

3) the fields are $\perp$ to each other and to the direction of propagation.

4) $\vec{B} = \frac{\hat{k} \times \vec{E}}{c}$ where $\hat{k}$ is the direction of propagation.

$$\vec{S} = (\vec{E} \times \vec{B})/\mu_0$$

energy density flow
(poynting vector)

$$\frac{dU}{dt} = -(\vec{\nabla}\cdot\vec{S}) = -\int \vec{S}\cdot d\vec{a}$$

Multipole Expansion

$$\phi_{(r)} = \frac{1}{4\pi\varepsilon_0}\left[\frac{1}{r}\int \rho\, dv' + \frac{1}{r^2}\int r'\cos\theta\, dv' + \dots\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{k_0}{r} + \frac{k_1}{r^2} + \frac{k^2}{r^3} + \dots\right]$$

$r \rightarrow$ position where Φ is being calculated

$r' \rightarrow$ position of charge element

$\theta \rightarrow$ angle between r & r'

$K_0 \rightarrow$ monopole moment

$K_1 \rightarrow$ dipole moment

$K_2 \rightarrow$ quadrapole moment.

Φ due to a dipole $= \dfrac{\hat{r} \cdot \vec{p}}{4\pi\varepsilon_0 r^2} = \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

$$\vec{E} = -\vec{\nabla}\Phi$$

$$E_r = \frac{p\cos\theta}{2\pi\varepsilon_0 r^2}, \quad E_\theta = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}$$

$\vec{\tau}$ on a dipole $= \vec{p} \times \vec{E}$

$\vec{F}$ on a dipole $= (\vec{p} \cdot \vec{\nabla})\vec{E}$

Polarization of an atom due a filed

$$\vec{p} = \alpha \vec{E}$$

$\alpha = 4\pi\varepsilon_0 a^3$

$\quad$ ↳ typical atomic radius.

Dielectrics

↳ Characterised by K.

↳ Polarization density $\underline{\vec{P}} = \vec{p} N = \varepsilon_0 (K-1) \vec{E}$

$\quad$ ↳ $\vec{p}$ dipole moment, N no dencity.

↳ $\chi_e = K - 1$

↳ Charge density on surface $\sigma = |\underline{\vec{P}}|$