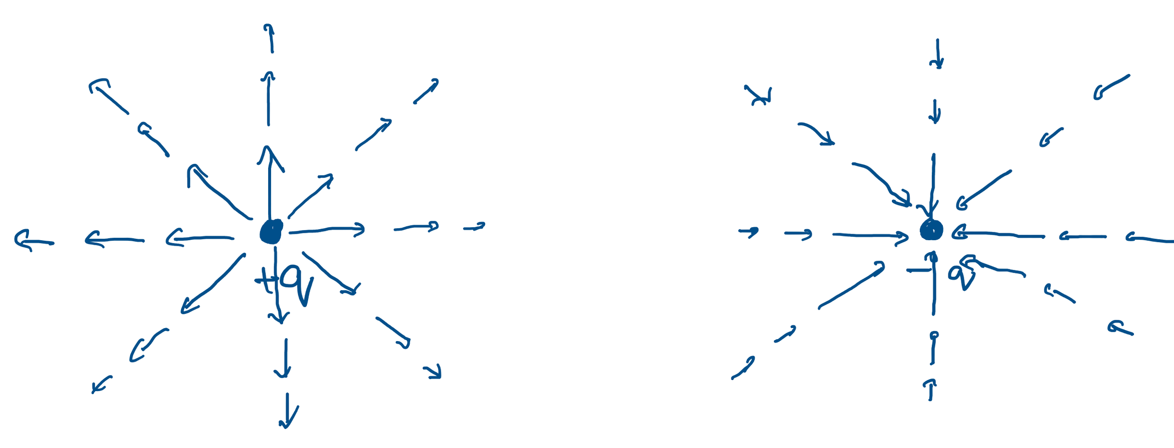
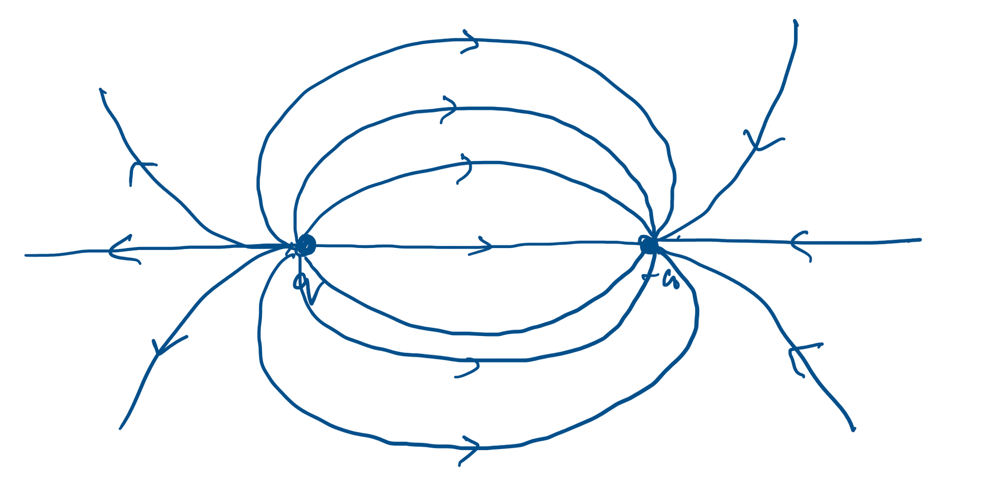
Lecture 9
Tuesday, 4 February 2020 11:52 PM

Let us define a new quantity related to charges that can be defined independently of a second charge.

$$\vec{E} = Kq/r_{12}^2 \; \hat{r}_{12}$$

electric field





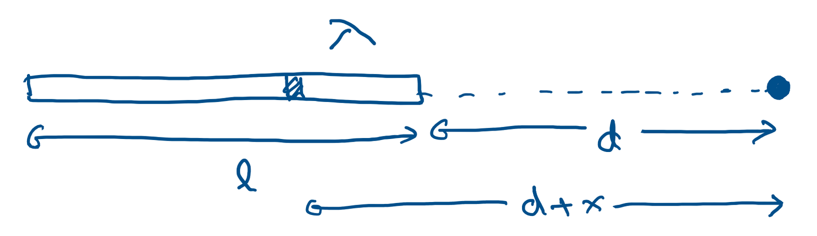
If we had multiple particles,

$$\vec{E} = \sum_{j=1}^{k} \frac{K q_j \hat{r}_{0j}}{r_{0j}^2}$$

For continuous charge distributions,

$$\vec{E} = \int \frac{K \rho \, dV}{r^2} \hat{r}$$

1) Find the $\vec{E}$ field for the following setup

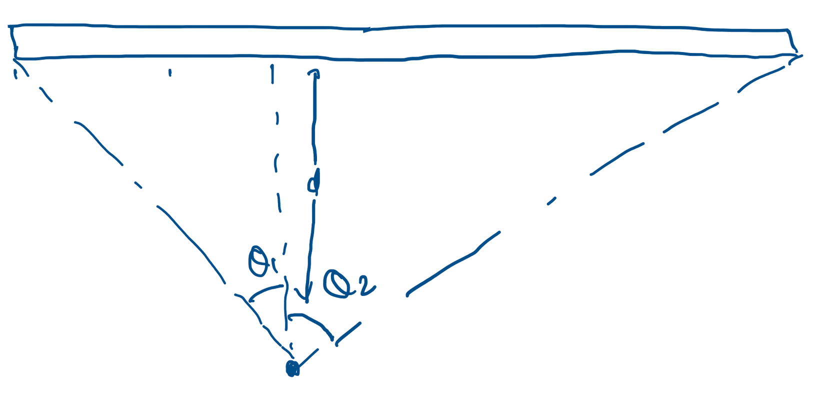


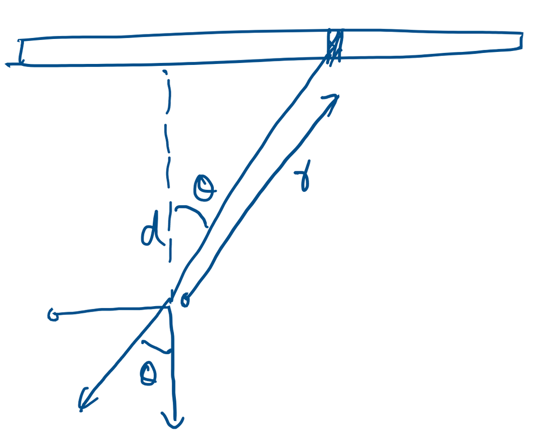
$$\vec{E} = \int_0^l \frac{K \lambda dx}{(d+x)^2}$$

$$= -\frac{K\lambda}{(d+x)} \Big|_0^l$$

$$= \frac{K\lambda}{d} - \frac{K\lambda}{d+l}$$

2)





$r\cos\theta = d$ | $x = r\sin\theta$
$\Rightarrow r = d/\cos\theta$ | $= d\tan\theta$

$$\therefore E_x = -\int_{-\theta_1}^{\theta_2} \frac{K\lambda\, dx}{r^2} \sin\theta$$

$$= -\frac{K\lambda}{d^2} \int_{-\theta_1}^{\theta_2} \cos^2\theta \sin\theta \, dx$$

$$= -\frac{K\lambda}{d^2} \int_{-\theta_1}^{\theta_1} \cancel{\cos^2\theta} \sin\theta \, d \, \cancel{\sec^2\theta} \, d\theta$$

$$= -\frac{K\lambda}{d} \int_{-\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$= \frac{K\lambda}{d} (\cos\theta_2 - \cos\theta_1)$$

$$E_y = -\int_{-\theta_1}^{\theta_2} \frac{K\lambda}{r^2} dx \cos\theta$$

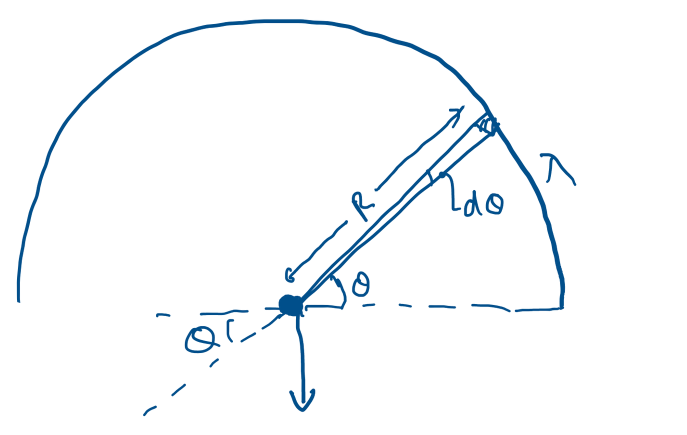
$$= -\int_{-\theta_1}^{\theta_2} \frac{K\lambda}{d^2} \cos^{\cancel{3}}\theta \, \cancel{\sec^2\theta} \, d\theta \, \cancel{d}$$

$$= -\frac{K\lambda}{d} (\sin\theta_2 + \sin\theta_1)$$

$\theta_1 = \theta_2 = \pi/2$

$$E_y = -\frac{2K\lambda}{d}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0 d}$$

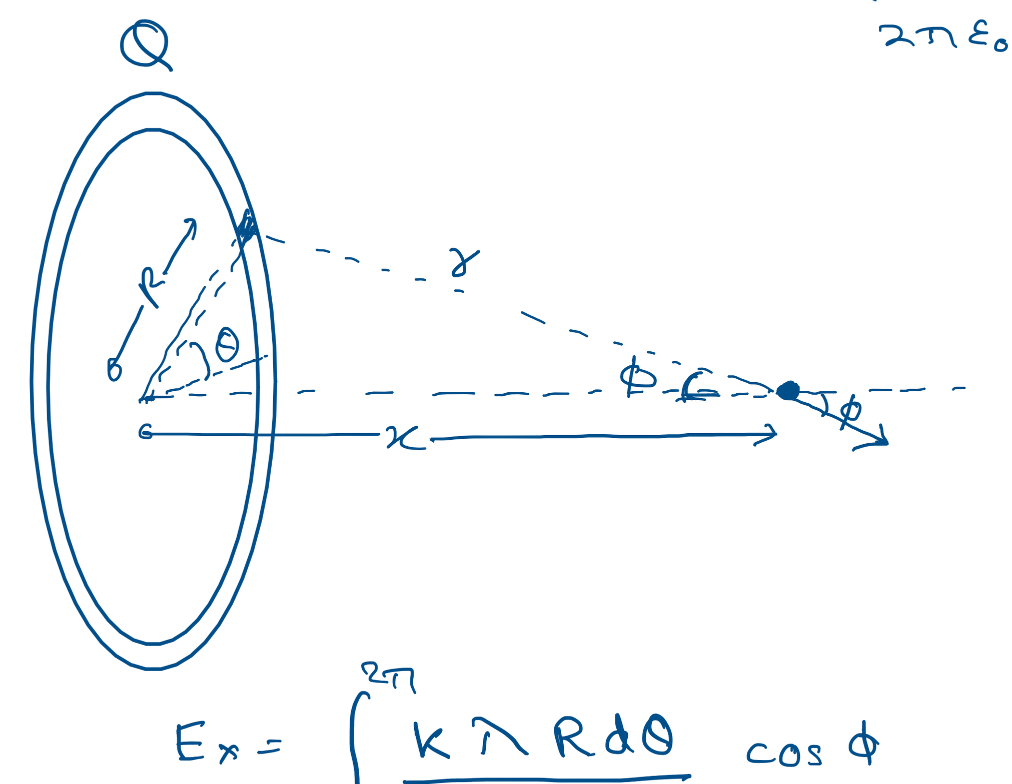


$$E_y = -\int \frac{K\lambda R d\theta}{R^2} \sin\theta$$

$$= -\int_0^{\pi} \frac{K\lambda\, d\theta}{R} \sin\theta$$

$$= \frac{K\lambda}{R} [\cos\pi - \cos\theta)$$

$$= -\frac{\lambda}{2\pi\varepsilon_0 R}$$

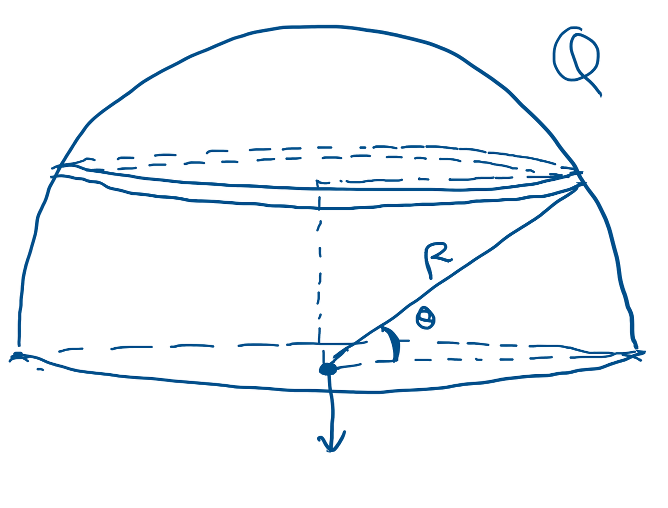


$$E_x = \int_0^{2\pi} \frac{K\lambda R d\theta}{(x^2+R^2)} \cos\phi$$

$$= \int_0^{2\pi} \frac{K\lambda R\, d\theta}{(x^2+R^2)^{3/2}} x$$

$$= \frac{K\lambda 2\pi R x}{(x^2+R^2)^{3/2}}$$

$$= \frac{KQx}{(x^2+R^2)^{3/2}}$$



$$\sigma = \frac{Q}{4\pi R^2}$$

$$dE_z = -\frac{K dQ (R\sin\theta)}{(R^2\sin^2\theta + R^2\cos^2\theta)^{3/2}}$$

$$= -\frac{K(\sigma 2\pi \cancel{R^2} d\theta) \cancel{R} \sin\theta}{\cancel{R^3}}$$

$$E_z = -\int_0^{\pi/2} K\sigma 2\pi \, d\theta \sin\theta$$

$$= K\sigma 2\pi \cos\theta \Big|_0^{\pi/2}$$

$$= -K\sigma 2\pi$$

$$= -\frac{KQ}{2R^2}$$