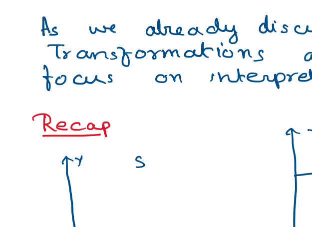
Lecture 3 — Velocity Raptor

Thursday, 16 January 2020 12:50 AM

As we already discussed in class how Lorentz Transformations are developed, we will mostly focus on interpreting & using them.

Recap



$$c\Delta t' = \gamma(c\Delta t - \beta\Delta x) \qquad c\Delta t = \gamma(c\Delta t' + \beta\Delta x')$$

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t) \qquad \Delta x = \gamma(\Delta x' + \beta c\Delta t')$$

To check that these transformations hold, lets make sure they agree with the fundamental relationships we had earlier obtained.

Loss of Simultaneity

Let us consider two events in S' s.t. $(\Delta t', \Delta x') = (0, \Delta x')$

$\therefore\ c\Delta t = \gamma\beta\Delta x'$

$\Delta t = \dfrac{\gamma\beta\Delta x'}{c}$ — If the events are separated in space, they will no longer be simultaneous.

Time Dilation

Consider two events in S' s.t. $(\Delta t', \Delta x') = (\Delta t', 0)$

$c\Delta t = \gamma c\Delta t'$

$\Delta t = \gamma\Delta t'$

As $\gamma \geq 1$, $\Delta t \geq \Delta t'$

Conversely, for S s.t. $(\Delta t, \Delta x) = (\Delta t, 0)$

$c\Delta t' = \gamma(c\Delta t)$

$\Delta t' = \gamma\Delta t$

As $\gamma \geq 1$, $\Delta t' \geq \Delta t$

This agrees with our previous analysis.

Length Contraction

To measure a length is S we need to do so simultaneously.

$\therefore (\Delta t, \Delta x) = (0, \Delta x)$

$\therefore\ \Delta x' = \gamma\Delta x$

$\therefore\ l = l'/\gamma$

Lets look at a few problems right away.

Problem 1

A train with proper length L moves with speed $5c/13$ with respect to the ground. A ball is thrown **from the back of the train to the front. The speed of the ball wrt the train is $c/3$. As viewed by someone from the ground, how much time does the ball spend in transit and how far does it travel?**

$\Delta x_T = L$

$\Delta t_T = \dfrac{L}{c/3}$

$\therefore\ \Delta x_G = \gamma(\Delta x_T + \beta\Delta t_T) = \dfrac{13}{12}\left(L + \dfrac{5}{13}3L\right)$

$= \dfrac{13}{12}\dfrac{28}{13}L$

$= 7/3\, L$

$\Delta t_G = \gamma\left(\dfrac{3L}{c} + \dfrac{5c}{13}\dfrac{L}{c^2}\right) = \dfrac{11L}{3c}$

Alternate method,

$\Delta t_G = \gamma\left(\Delta t_T + \dfrac{Lv}{c^2}\right) = \dfrac{11L}{3c}$

& $\Delta x_G = L/\gamma + v\Delta t_G = 7L/3$

In class we also discussed about velocity addition, which goes as

$$w = \frac{u + v}{1 + \frac{uv}{c^2}}$$

u → relative velocity of frames

v → velocity in S' frame

Here, we have assumed S' moves with $+u$ in the x-direction

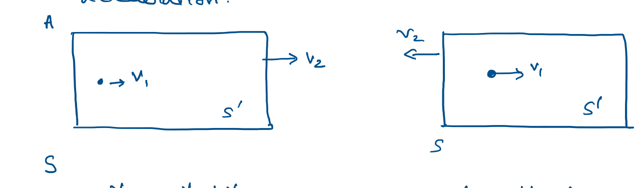
If $v_1, v_2 \ll c^2$, then

$w = u + v$

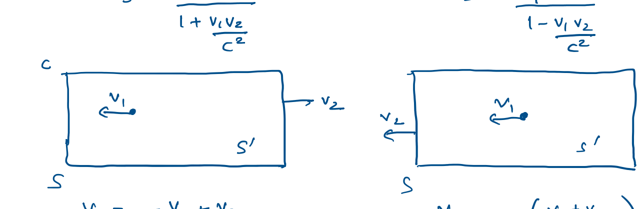
If $u = c$ or $v = c$

then $w = c$

If you add any two velocities less than c, the result will also be less than c. We will see later that this is why we can never achieve the speed of light even with constant acceleration.



$v_s = \dfrac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}}$ $\qquad$ $v_s = \dfrac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}$



$v_s = \dfrac{-v_1 + v_2}{1 - \frac{v_1 v_2}{c^2}}$ $\qquad$ $v_s = -\left(\dfrac{v_1 + v_2}{1 + v_1 v_2/c^2}\right)$

Problem 2

A and B travel at $4c/5$ and $3c/5$ respectively. How fast should C travel between them, so that C sees A & B approach her at the same speed? What is this speed?

Suppose C is travelling with v.

$V_{AC} = \dfrac{4c/5 - v}{1 - \frac{4v}{5c}}$

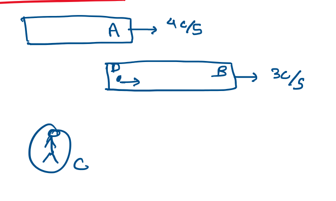
& $V_{BC} = \dfrac{3c/5 - v}{1 - \frac{3v}{5c}}$

$\therefore$ If speed of approach is same,

$$\frac{4c/5 - v}{1 - \frac{4v}{5c}} = \frac{3c/5 - v}{1 - \frac{3v}{5c}}$$

Using Mathematica, we get $5/7\, c$

Problem 3



i) How long as viewed by A and as viewed by B, does it take to overtake B?

ii) Let event 1 be "the front of A passing the back of B" and event 2 be "the back of A passing the front of B". Person D walks from the back of B to the front s.t. they coincide with both events E_1 & E_2. How long does the overtaking take as viewed by D?

i) From B's point of view,

$V_A = \dfrac{4c/5 - 3c/5}{1 - \frac{4}{5}\frac{3}{5}} = \dfrac{5c}{13}$

Length to overtake $= L_B + L_A$

$= L + \dfrac{L}{\gamma}$

$= 25L/13$

$\therefore$ Time to overtake $= \dfrac{25L/13}{5c/13} = 5L/c$

The exact same result will be obtained for A.

b) In D's frame,

D is at rest and the trains approach D with equal speed to ensure that E_2 coincide with D's location. Let this speed be v.

$\therefore\ \dfrac{v + v}{1 + v^2/c^2} = \dfrac{5c}{13}$ } found from part a

$\Rightarrow 5\beta^2 - 26\beta + 5 = 0$

$\Rightarrow \beta = 1/5$ we ignore $\beta = 5$

$\therefore\ \gamma = \dfrac{5}{2\sqrt{6}}$

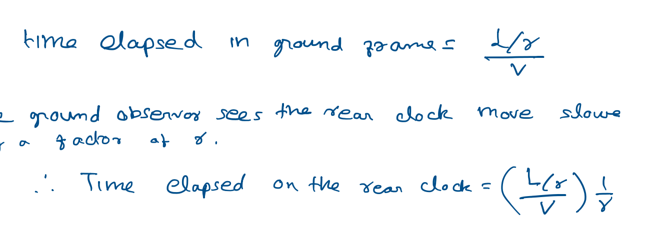
$\therefore\ t_D = \dfrac{L/\gamma}{c/5} = \dfrac{2\sqrt{6}L}{c}$

Problem 4

a) Two trains with proper length L and travel on parallel tracks. They both move with speed 'v' with respect to the ground, one rightward and one leftward. The clocks at the front of the train both read zero when the fronts coincide. What do the clocks at the back of the trains read when the backs coincide? Answer using ground frame.

b) Repeat the same in B's frame. Consider $v = c/2$.

a)



After each train travels a distance L/γ, the backs coincide at the same location as the fronts.

$\therefore$ time elapsed in ground frame $= \dfrac{L/\gamma}{v}$

The ground observer sees the rear clock move slower by a factor of γ.

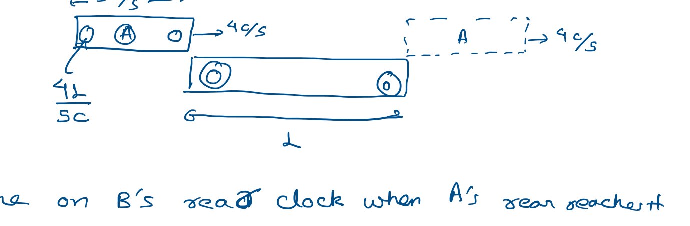
$\therefore$ Time elapsed on the rear clock $= \left(\dfrac{L/\gamma}{v}\right)\dfrac{1}{\gamma}$

$\therefore$ Final reading $= \dfrac{Lv}{c^2} + \dfrac{L}{\gamma^2 v} = \dfrac{L}{v}$

b) In B's frame,

$V = \dfrac{c/2 + c/2}{1 + \frac{1}{2}\frac{1}{2}} = \dfrac{4c}{5}$

$\therefore$



Time on B's rear clock when A's rear reaches it

$0 + \dfrac{3L/5 + L}{4c/5} = 2L/c$ — agrees with below

Time elapsed for A's rear clock $= \left(\dfrac{2L}{c}\right)\dfrac{1}{\gamma_{4/5}}$

$\therefore$ Reading $= \dfrac{4L}{5c} + \dfrac{2L}{c}\dfrac{3}{5} = \dfrac{2L}{c}$