Lecture 2

Monday, 13 January 2020 9:37 AM

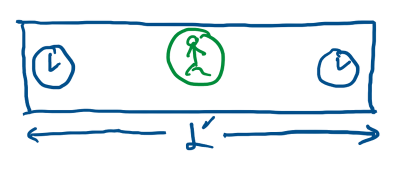
Recap

- Two events that are simultaneous in one frame are not necessarily simultaneous in another frame. The only exception is that if two events occur at the same time and location in one frame. They will then be simultaneous in all frames.
- If two objects move past each other, they both see each other's clocks moving slowly by a factor γ in their own frame.

(Lets see how to resolve that)

Head Start

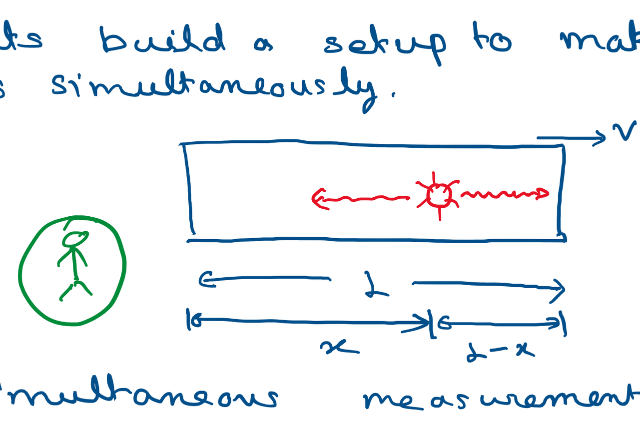
In the frame of the train, we have two synchronized clocks



If the tram passes by a person on the ground at speed 'v', how will they read the clocks?

"To compare two clocks they must be measured simultaneously"

So lets build a setup to make sure we measure the clocks simultaneously.



For simultaneous measurement

$$\frac{x}{c+v} = \frac{L-x}{c-v}$$

$$\Rightarrow x(c-v) = L(c+v) - x(c+v)$$

$$\Rightarrow x = \frac{L(c+v)}{2c}$$

$$\& \quad L - x = \frac{L(c-v)}{2c}$$

Comparing in the train's frame, light travels an extra distance of $\frac{L'(c+v)}{2c} - \frac{L'(c-v)}{2c} = \frac{L'v}{c}$

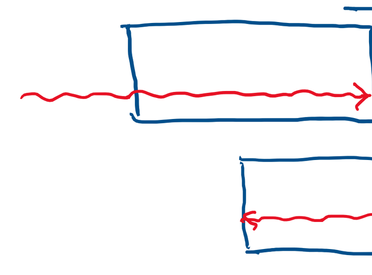
$\therefore$ Light takes an extra time to travel $\frac{L'v}{c^2}$

And the ground observer notes two unsynchronized clocks with a separation in time of $L'v/c^2$

Points to note:

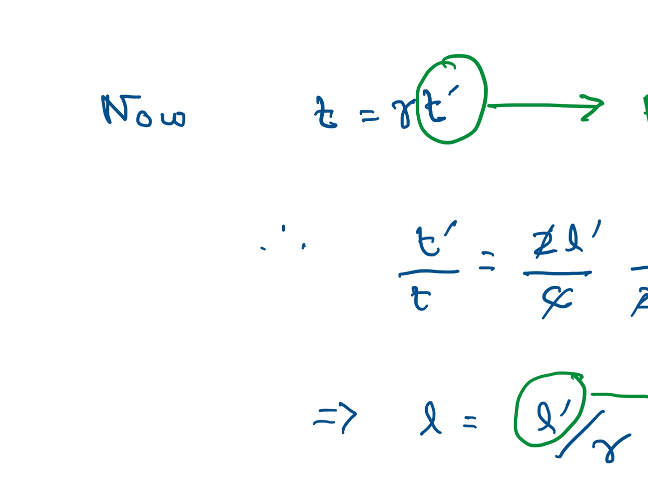
1) L' is the length of the train in it's own frame.
2) The two clocks still tick at the same rate. They are just unsynchronized by a constant offset.

Length Contraction



$$t' = \frac{2l'}{c}$$

Ground frame



$$t = \frac{l}{c-v} + \frac{l}{c+v}$$

$$= \frac{2lc}{c^2 - v^2}$$

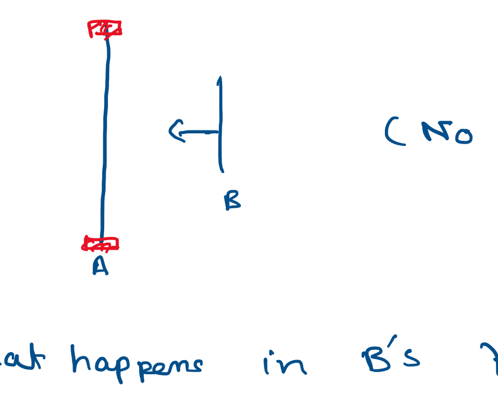
$$= \frac{2l}{c}\gamma^2$$

Now $t = \gamma t'$ → proper time (shortest of all frames)

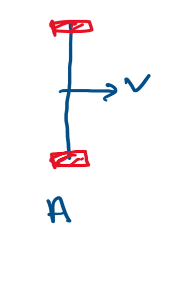
$$\therefore \frac{t'}{t} = \frac{\cancel{2}l'}{\cancel{c}} \frac{\cancel{c}}{\cancel{2}l\gamma^2}$$

$$\Rightarrow l = l'/\gamma$$ → proper length (longest of all frames)

Transverse length contraction

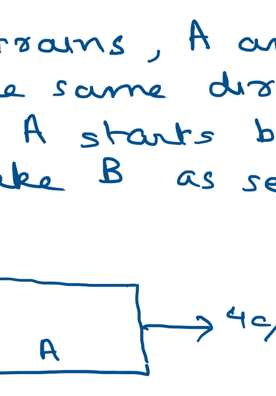


What happens in A's frame?



(No marks on B)

What happens in B's frame?

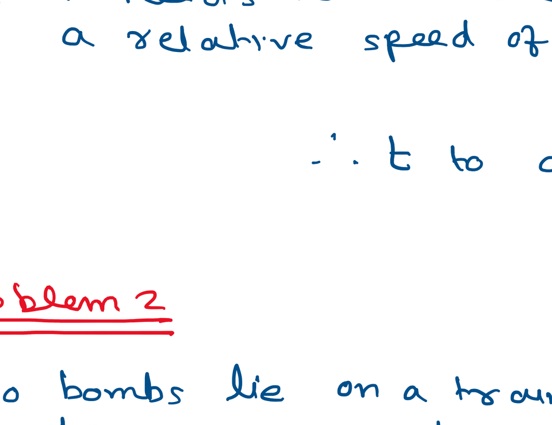


This leads to a contradiction.

$\therefore$ transverse contractions do not occur.

Problem 1

Two trains, A and B, each have proper length L and move in the same direction. A's speed is $4/5\,c$ & B's speed is $3/5\,c$. A starts behind B. How long does A take to overtake B as seen from the ground?



$\gamma_A = 5/3$, $\gamma_B = 5/4$

$\therefore L_A = 3/5\,L$

$L_B = 4/5\,L$

$\therefore$ A needs to overtake B by crossing $7/5\,L$ with a relative speed of $c/5$

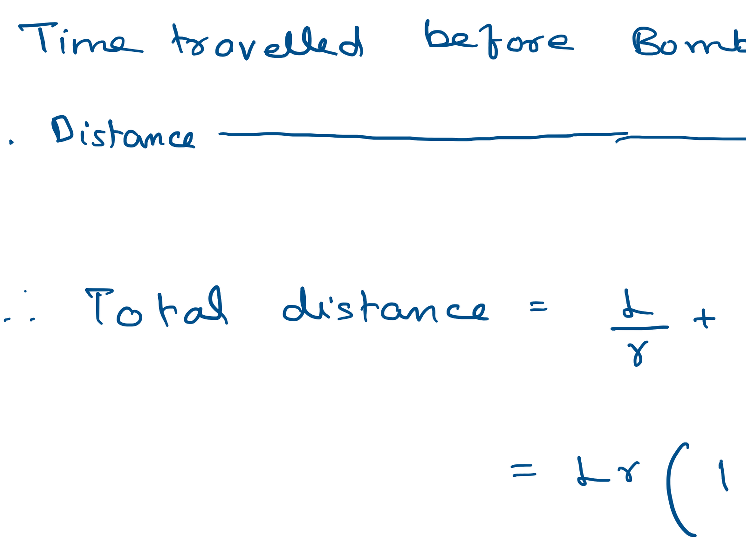
$\therefore$ t to overtake $= 7L/c$

Problem 2

Two bombs lie on a train platform at a distance L apart. As a train passes by at speed v, the bombs explode simultaneously and leave marks on the train. Due to length contraction of the train, the marks on the train must be γL apart when viewed in train's frame.

How would someone on the tram explain the marks are γL apart considering the bombs were L/γ apart in their frame?

The resolution lies in the fact that the explosions are not simultaneous in the train's frame.
Let the clocks read time 't' when they explode.
$\therefore$ in train's frame



Time travelled before Bomb 1 explodes $= \left(Lv/c^2\right)\gamma$

$\therefore$ Distance $= \left(\frac{Lv^2}{c^2}\right)\gamma$ ← time dilation

$$\therefore \text{Total distance} = \frac{L}{\gamma} + L\frac{v^2}{c^2}\gamma$$

$$= L\gamma\left(1 - \cancel{\frac{v^2}{c^2}} + \cancel{\frac{v^2}{c^2}}\right)$$

$$= L\gamma$$

Problem 3

A sailboat is manufactured so that the mast leans at an angle θ' with respect to the deck. An observer standing on a dock sees the boat go by a speed v. What angle does the observer see the mast make?

To an observer on the boat, height of mast $= l'\sin\theta'$
horizontal length $= l'\cos\theta'$

To an observer on the deck, the horizontal length is contracted to $\frac{l'\cos\theta'}{\gamma}$

$$\therefore \tan\theta = \frac{l'\sin\theta'}{l'\cos\theta'/\gamma} = \gamma\tan\theta'$$

$$\theta = \tan^{-1}\left(\gamma\tan\theta'\right)$$