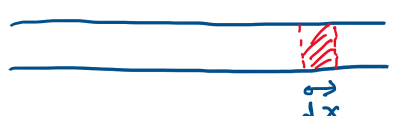
Lecture 17

Wednesday, 4 March 2020 9:47 PM

Current



$I = \frac{dq}{dt}$ ← amount of charge flowing through a surface in unit time

$$I = \frac{n\, A dx\, e}{dt}$$

$$= ne A V_d$$

↖ drift velocity.

$\vec{J}$ = Current Density = $I/A = ne\,\vec{v}_d$

Proper definition $I = \int \vec{J}\cdot d\vec{a}$

Furthermore, $\frac{\partial \rho}{\partial t} + \vec{\nabla}\cdot\vec{J} = 0$ ← conservation of charge.

for steady state current flow $\vec{\nabla}\cdot\vec{J} = 0$

Ohm's Law

$\vec{J} = \sigma \vec{E} \iff V = IR$ where $R = \rho \frac{L}{A}$

ρ = resistivity = $\frac{1}{\sigma}$

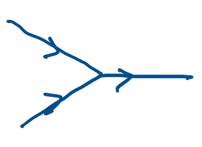
Finally, the power dissipated across any circuit element

$P = VI$

for ohmic elements ⇒ $P = I^2 R$

Circuits

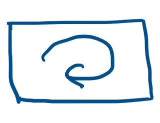
1). Charge conservation. $\vec{\nabla}\cdot\vec{J} = 0$



2) Kirchoff's Loop Law

↳ Electrostatic Field

↳ $\oint \vec{E}\cdot d\vec{s} = 0$



(*) In a 6 GeV electron synchotron, electrons travel around the machine in an approximately circular path 240 meters long. It is normal to have about 10^{11} e's circling on this path during a cycle of acceleration. The speed of the electrons is practically that of light. What is current?

No of trips made per second = $\frac{3\times10^8 \text{ m/s}}{240} = 1.25\times10^6\ s^{-1}$

$\therefore I = \frac{Q}{T}$ ← charge through loop per sec; T ← 1sec

$= 1.25\times10^6 \times 10^{11} \times 1.6\times10^{-19}$ A

$= 0.02$ A

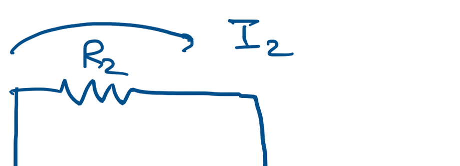
(*) We have 5×10^{16} doubly charged positive ions per m^3, all **moving west with a speed of 10^5** m/s. In the same region there are 10^{17} e per m^3 moving northeast with a speed of 10^6 m/s. What is the magnitude and direction of J?

$J_{ion} = neV_d = 2e\ 5\times10^{16}\times10^5 = 1.6\times10^3\ A/m^2\ (-\hat{i})$

$J_{elec} = nev_d = e\times10^{17}\times10^6 = -1.6\times10^4\ A/m^2\ (\hat{i}+\hat{j})$

$\vec{J}_{tot} = \left\langle -J_{ion} - \frac{J_{elec}}{\sqrt{2}}, -\frac{J_{elec}}{\sqrt{2}} \right\rangle$

(*)



$V_{tot} = IR_1 + IR_2$ ← drop in voltage

$V_{tot} = I R_{eq}$

$\therefore R_{eq} = R_1 + R_2$

$\varepsilon - I_1R_1 = 0$

$I = I_1 + I_2$

$-I_2R_2 + I_1R_1 = 0$

$\Rightarrow I = I_1 + \frac{I_1R_1}{R_2}$

$\Rightarrow I = \frac{\varepsilon}{R_1} + \frac{\varepsilon}{R_2}$

$\Rightarrow \varepsilon = I\frac{R_1R_2}{R_1+R_2}$

$R_{eff} = \frac{R_1R_2}{R_1+R_2}$

(*)

$-\frac{Q}{C_1} - \frac{Q}{C_2} + \varepsilon = 0$

$\varepsilon = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$

Comparing with $\varepsilon = Q/C_{eq}$

$\therefore C_{eq} = \frac{1}{1/C_1 + \frac{1}{C_2}}$

$= \frac{C_1C_2}{C_1+C_2}$

$\varepsilon = \frac{Q_1}{C_1}$

$Q = Q_1 + Q_2$

$-\frac{Q_1}{C_1} + \frac{Q_2}{C_2} = 0$

$\Rightarrow Q = Q_1 + Q_1\frac{C_2}{C_1}$

$\Rightarrow Q = \varepsilon C_1 + \varepsilon C_2$

$\Rightarrow \varepsilon = \frac{Q}{C_1+C_2}$

(*)

$0 = \varepsilon - Q/C - IR$

$\Rightarrow \frac{dQ}{dt} = \frac{\varepsilon}{R} - \frac{Q}{RC}$

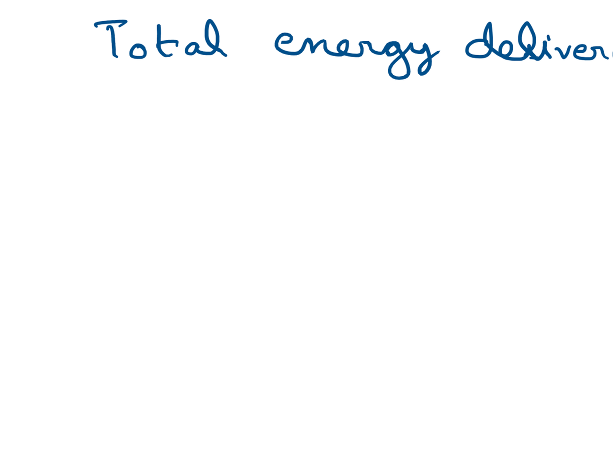
$\Rightarrow \frac{dQ}{\left(\varepsilon/R - Q/RC\right)} = dt$

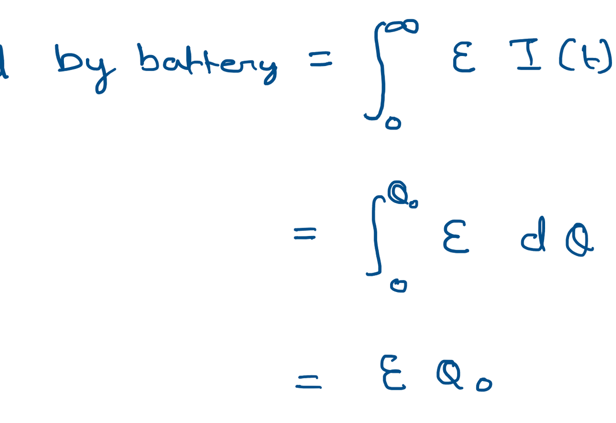
$\Rightarrow (-RC)\ln\left(\frac{\varepsilon/R - Q/RC}{\varepsilon/R}\right) = t$

$\Rightarrow \frac{\varepsilon}{R} - Q/RC = \varepsilon/R\ e^{-t/RC}$

$\Rightarrow Q = \varepsilon C\left(1 - e^{-t/RC}\right)$

& $I = \varepsilon/R\ e^{-t/RC}$





Total energy delivered by battery $= \int_0^\infty \varepsilon\, I(t)\, dt$

$= \int_0^{Q_0} \varepsilon\, dQ$

$= \varepsilon Q_0$

$= C\varepsilon^2$

$W_{resistor} = \int_0^\infty I^2(t) R\, dt$

$= \int_0^\infty \frac{\varepsilon^2}{R} e^{-2t/RC}\, dt$

$= \frac{1}{2} RC \frac{\varepsilon^2}{R} = \frac{1}{2}C\varepsilon^2$

$W_{capacitor} = \frac{1}{2}\frac{Q^2[\infty]}{C}$

$= \frac{1}{2}C\varepsilon^2$