Lecture 15

Wednesday, 26 February 2020 3:44 PM

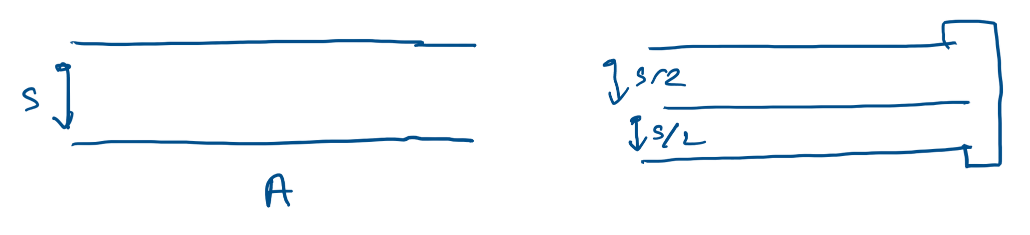
Capacitance

1) Draw 2 conductors of interest in the absence of $\vec{E}$ & charges
2) Give One conductor $+Q$ & the other $-Q$
3) Calculate ΔV between the conductors
4) $C = Q/V$ or ΔV

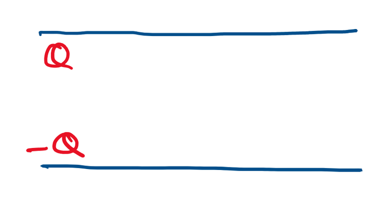
→ For a single conductor, assume the other conductor's surface is at infinity.

→ Capacitance is a property of geometry of the conductors

⊛ Find the capacitance for.



a)



As the $\vec{E}$ inside the conductors are zero, the Q charges spread out evenly on the inner surfaces

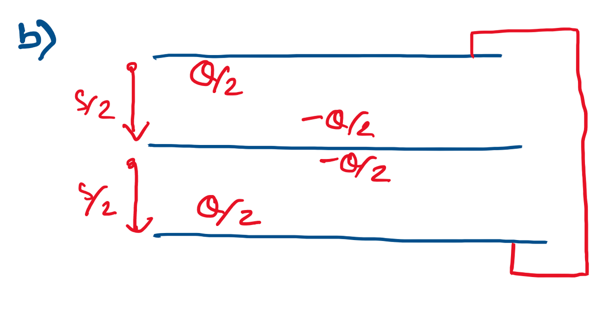
$\therefore \sigma = Q/A$

& $\vec{E} = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$

$\therefore \Delta V = \dfrac{Qs}{A\varepsilon_0}$

$\Rightarrow C = Q/\Delta V = \varepsilon_0 A/s$

b)



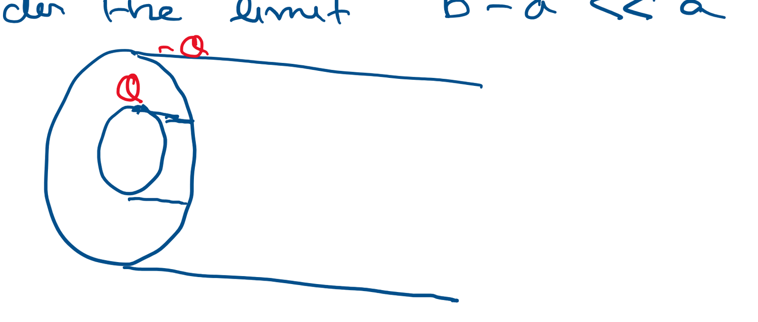
$\tilde{\sigma} = Q/2A$

$E = Q/2A\varepsilon_0$

$\tilde{V} = E \times s/2 = Qs/4A\varepsilon_0$

$\therefore \tilde{C} = Q/\tilde{V} = \dfrac{4A\varepsilon_0}{s} = 4C$

⊛ A capacitor consists of two coaxial cylinders of length L, inner and outer radii a and b. Assume $L \gg b-a$. Ignore end corrections. Find C and consider the limit $b - a \ll a$



$\vec{E}$ inside the two conductors $= \dfrac{Q}{2\pi r L \varepsilon_0}\hat{r}$

$$\Delta V = \left|\int \vec{E}\cdot d\vec{s}\right|$$

$$= \int_a^b \frac{Q}{2\pi r L \varepsilon_0}\,dr$$

$$= \frac{Q}{2\pi L\varepsilon_0}\ln\left(b/a\right)$$

$$\therefore C = Q/V = \frac{2\pi L\varepsilon_0}{\ln(b/a)}$$

For $b = a+s$, $s \ll 1$

$$\therefore C = \frac{2\pi L\varepsilon_0}{\ln\left(\frac{a+s}{a}\right)}$$

$$= \frac{2\pi L\varepsilon_0}{\frac{s}{a} + O\left[s/a\right]^2}$$

$$= \frac{2\pi a L\varepsilon_0}{s}$$

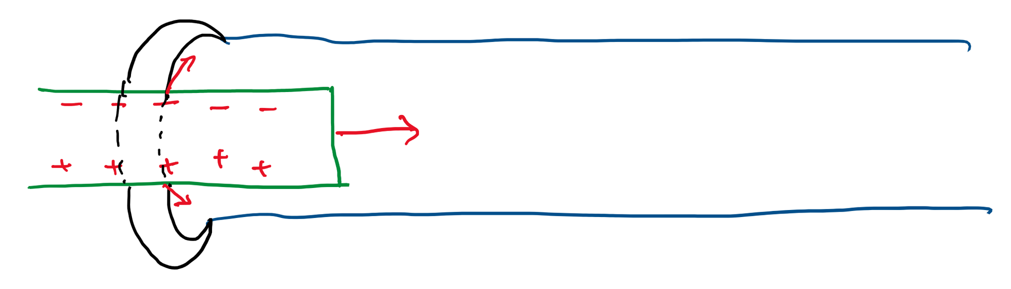
$$= \varepsilon_0 A/s$$

Conductor in a Capacitor

a) The plates of a capacitor have area A and separation A. The plates are isolated so that charge on them remain constant. ($\pm\sigma$). A neutral conducting slab with area A and thickness $s/2$ is initially held outside the capacitor. The slab is released. What is the K.E. at the moment it is completely inside the capacitor.

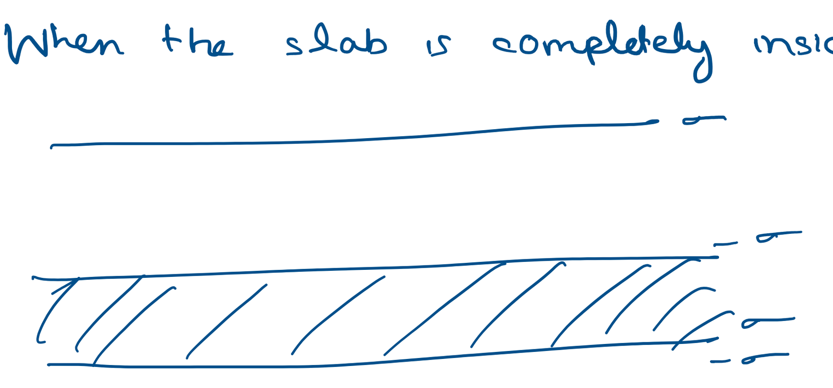
b) Repeat the problem for the case with capacitors connected to a battery with fixed voltage.

Comment on edge effect



a) $U_i = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{\sigma^2 As}{2\varepsilon_0}$

When the slab is completely inside the conductor,



$U_f = \dfrac{1}{2}\varepsilon_0 E^2 = \dfrac{\sigma^2 As}{4\varepsilon_0}$

$\therefore K = U_i - U_f = \dfrac{\sigma^2 As}{4\varepsilon_0}$

b)

$U_i = \dfrac{\sigma^2 As}{2\varepsilon_0}$

V_i (potential difference) $= \dfrac{\sigma s}{\varepsilon_0}$

in terms of $\vec{E}$, $\Delta V = E\,d$
↑ distance between plates

$\therefore E_f = \dfrac{\sigma s}{\frac{s}{2}\varepsilon_0} = \dfrac{2\sigma}{\varepsilon_0}$

$\therefore U_f = \dfrac{\varepsilon_0}{2}\left(\dfrac{2\sigma}{\varepsilon_0}\right)^2 \dfrac{As}{2}$

$= \dfrac{\sigma^2 As}{\varepsilon_0}$

But this means we have more energy than we started with. How can we find out K.

Extra Charge needed to maintain Potential $= \sigma A$

$\therefore$ Energy Spent by battery $= \sigma A \dfrac{\sigma s}{\varepsilon_0}$

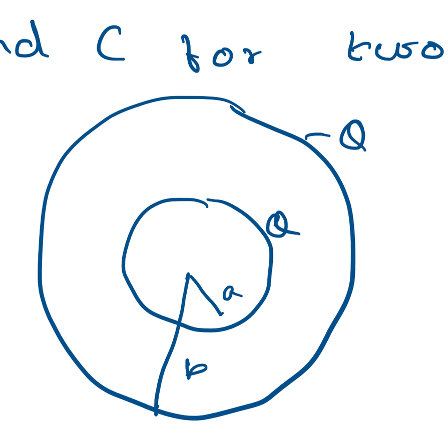
$= \dfrac{\sigma^2 As}{\varepsilon_0}$

$\therefore$ KE $= U_i - U_f +$ Energy supplied

$= \dfrac{\sigma^2 As}{2\varepsilon_0}$

⊛ For completeness

Find C for two concentric spheres of radius a&b



$V(b) = 0$

$V(a) = -\dfrac{kQ}{b} + \dfrac{kQ}{a}$

$= kQ\left[\dfrac{1}{a} - \dfrac{1}{b}\right]$

$\therefore C = Q/V$

$= \dfrac{4\pi\varepsilon_0}{\left[1/a - 1/b\right]}$

for $b \to \infty$

$C = 4\pi\varepsilon_0 a$