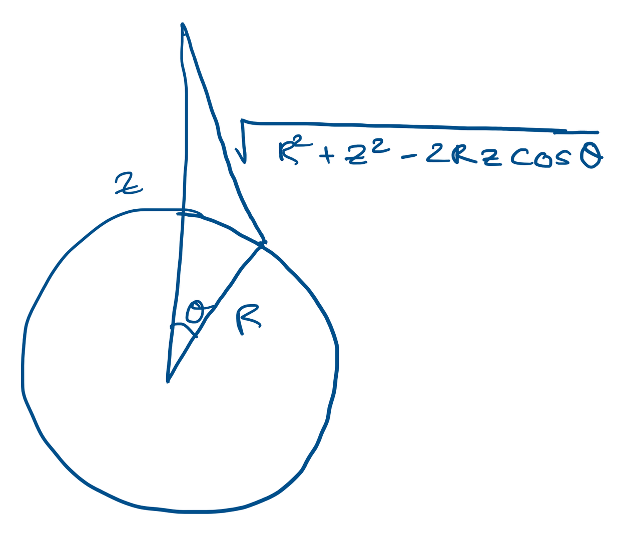
Lecture 13

Thursday, 20 February 2020 9:04 AM

⊛ Find the potential for a spherical shell using the above formula?



$$V = \int_0^{2\pi}\int_0^{\pi} \frac{k\sigma R^2 \sin\theta \, d\theta \, d\phi}{(R^2+z^2-2Rz\cos\theta)^{1/2}}$$

$$= 2\pi k\sigma R^2 \int_0^{\pi} \frac{\sin\theta \, d\theta}{(R^2+z^2-2Rz\cos\theta)^{1/2}}$$

$$= \frac{2\pi k\sigma R^2}{Rz} \left\{ R^2+z^2-2Rz\cos\theta \right\}^{1/2} \Big|_{\theta=0}^{\theta=\pi}$$

$$= \frac{2\pi k\sigma R}{z} \left\{ (R+z) - |R-z| \right\}$$

for $R < z$

$$V = \frac{4\pi k\sigma R^2}{z} = \frac{kQ}{z}$$

for $R > z$

$$V = \frac{2\pi k\sigma R \, 2\cancel{z}}{\cancel{z}}$$

$$= 4\pi R\sigma k$$

$$= kQ/R$$

⊛ Earlier we had found that $\vec{E}$ at the center of hemispherical shell of radius R and charge density σ

$$E = \pi k\sigma$$

Re-derive the result using Electric Potential V.

$$V = \int_0^{2\pi}\int_0^{\pi/2} \frac{k\sigma R^2 \sin\theta \, d\theta \, d\phi}{(R^2+z^2-2Rz\cos\theta)^{1/2}}$$

$$= 2\pi k\sigma R^2 \int_0^{\pi/2} \frac{\sin\theta \, d\theta}{(R^2+z^2-2Rz\cos\theta)^{1/2}}$$

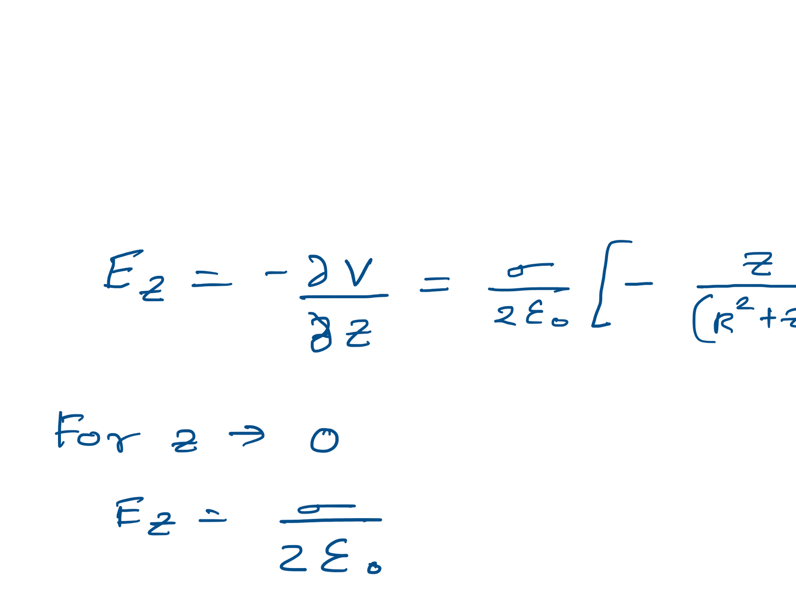
for $z \ll R$

$$\simeq 2\pi k\sigma R^2 \int_0^{\pi/2} \frac{(\sin\theta \, d\theta)}{R} \left(1+\frac{z}{R}\cos\theta\right)$$

$$= 2\pi k\sigma R \left[1+\frac{z}{2R}\right] = k\sigma\pi(2R+z)$$

$$\text{for } \vec{E} = -\frac{\partial V}{\partial z} = -k\sigma\pi$$

⊛ Similarly, find $\vec{E}$ for disc with surface charge density σ and radius R at distance z.



$$\phi = \int_0^R \frac{k\sigma \, 2\pi r \, dr}{(r^2+z^2)^{1/2}}$$

$$= \int_0^R \frac{k\sigma \, dx}{(x+z^2)^{1/2}}$$

$$= 2k\pi\sigma\left[(R^2+z^2)^{1/2} - z\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[(R^2+z^2)^{1/2} - z\right]$$

$$E_z = -\frac{\partial V}{\partial z} = \frac{\sigma}{2\varepsilon_0}\left[-\frac{z}{(R^2+z^2)^{1/2}} + 1\right]$$

For $z \to 0$

$$E_z = \frac{\sigma}{2\varepsilon_0}$$

⊛ We have two metal sphere of Radii R_1 and R_2 far apart from one another compared to their Radii. Divide Q between the spheres to minimize this potential energy

$$U = \underbrace{\frac{kq_1^2}{2R_1}}_{\frac{1}{2}\int \rho\phi \, dv} + \frac{k(Q-q_1)^2}{2R_2}$$

$$\frac{\partial V}{\partial q_1} = 0 = \frac{kq_1}{R_1} - \frac{k(Q-q_1)}{R_2}$$

$$\Rightarrow q_1/R_1 = (Q-q_1)/R_2$$

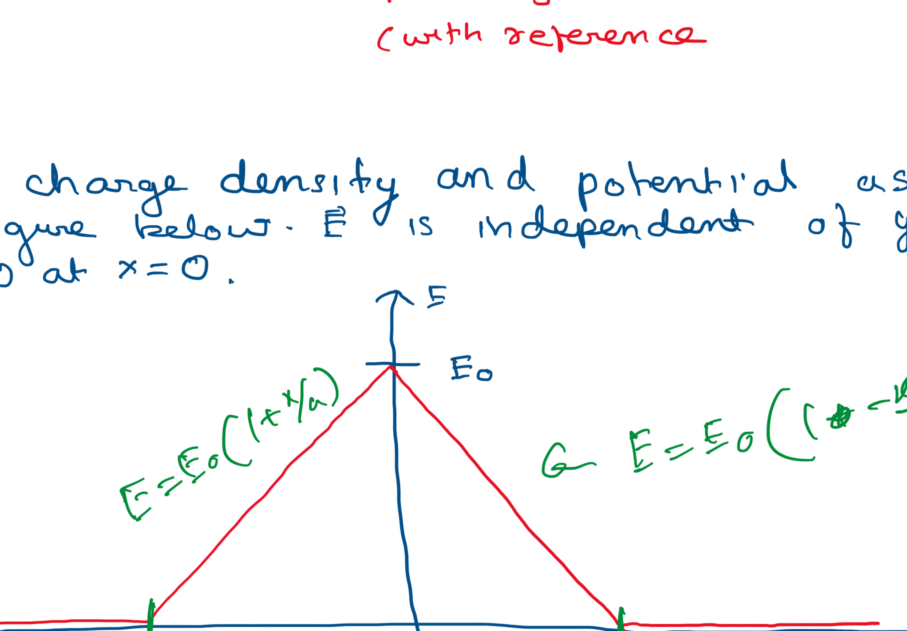
$$\Rightarrow q_1\left(\frac{R_1+R_2}{R_1R_2}\right) = Q/R_2$$

$$\Rightarrow q_1 = \frac{QR_1}{R_1+R_2}$$

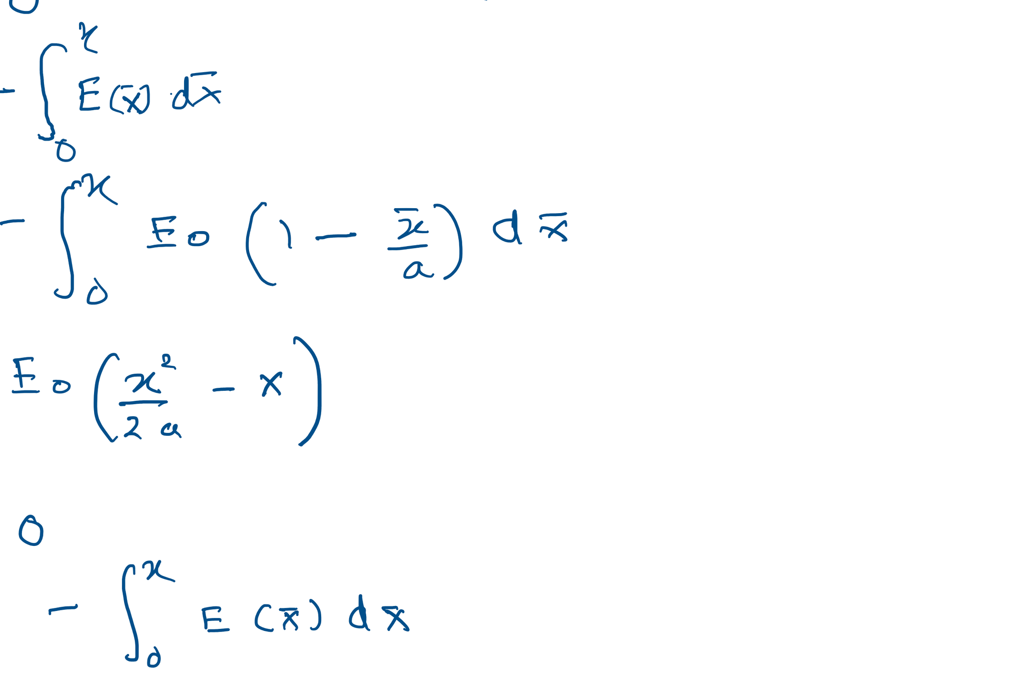
(do check for second derivative)

— x —

Instead of discussing on divergence and curl more than what has been taught in the class, we will just review all that we have learnt so far.



⊛ Find the charge density and potential associated to the $\vec{E}$ in the figure below. $\vec{E}$ is independent of y and z. Assume that $\varphi = 0$ at $x = 0$.



For $x > 0$

$$\phi = -\int_0^x E(\bar{x}) \, d\bar{x}$$

$$= -\int_0^x E_0\left(1-\frac{\bar{x}}{a}\right) d\bar{x}$$

$$= E_0\left(\frac{x^2}{2a} - x\right)$$

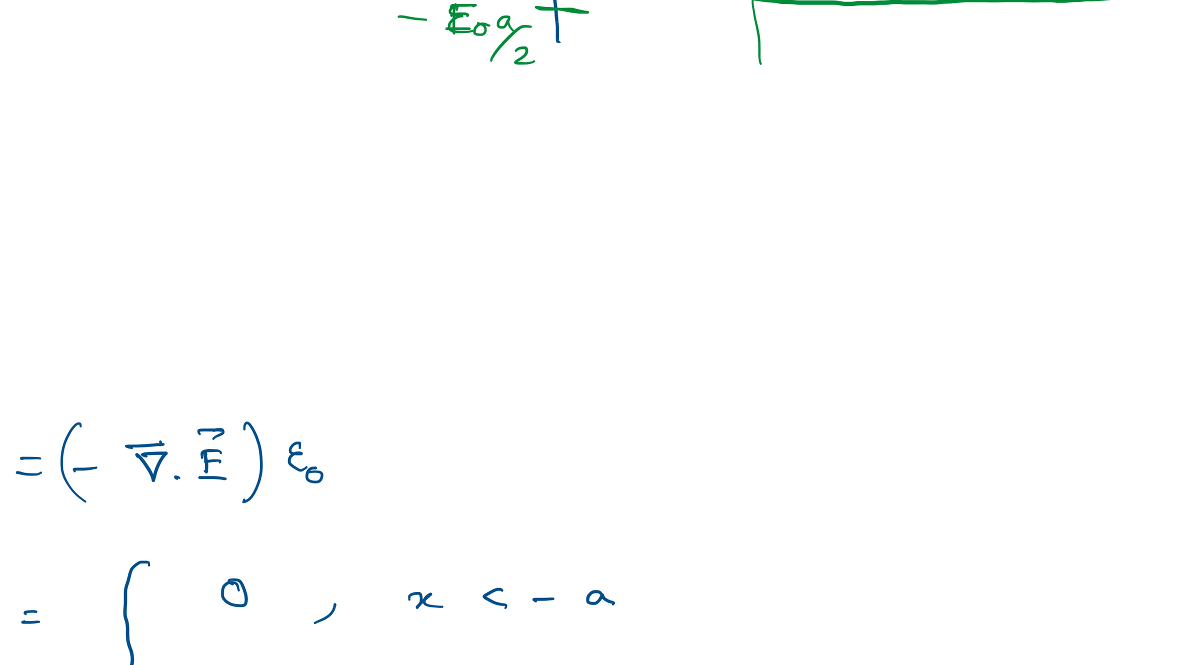
For $x < 0$

$$\phi = -\int_0^x E(\bar{x}) \, d\bar{x}$$

$$= -\int_0^x E_0\left(1+\frac{\bar{x}}{a}\right) d\bar{x}$$

$$= -E_0\left(\frac{x^2}{2a} + x\right)$$

$$\phi = \begin{cases} E_0 a/2 & , x < -a \\ -E_0\left(x^2/2a + x\right) & , -a \le x < 0 \\ E_0\left(x^2/2a - x\right) & , 0 \le x < a \\ -E_0 a/2 & , a \le x \end{cases}$$



$$\rho = \left(-\vec{\nabla}\cdot\vec{E}\right)\varepsilon_0$$

$$\rho = \begin{cases} 0 & , x < -a \\ \varepsilon_0 E_0/a & , -a \le x < 0 \\ -\varepsilon_0 E_0/a & , 0 \le x < a \\ 0 & , a \le x \end{cases}$$