# Formula Sheet 2

Thursday, 20 February 2020 4:07 PM

$$\vec{\nabla} = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)$$

$$\vec{\nabla} T = \left(\partial T/\partial x\,\hat{i} + \partial T/\partial y\,\hat{j} + \partial T/\partial z\,\hat{k}\right)$$

↳ scalar function

$$\therefore \vec{\nabla}\cdot\vec{V} = \left(\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}\right)$$

$$\& \quad \vec{\nabla}\times\vec{V} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}$$

## A few identities

i) $\vec{\nabla}(f+g) = \nabla f + \nabla g$

ii) $\vec{\nabla}\cdot(\vec{A}+\vec{B}) = \vec{\nabla}\cdot\vec{A} + \vec{\nabla}\cdot\vec{B}$

iii) $\vec{\nabla}\times(\vec{A}+\vec{B}) = (\vec{\nabla}\times\vec{A}) + (\vec{\nabla}\times\vec{B})$

iv) $\vec{\nabla}(fg) = f\,\vec{\nabla}g + g\,\vec{\nabla}f$

v) $\vec{\nabla}\cdot(f\vec{A}) = f(\vec{\nabla}\cdot\vec{A}) + \vec{A}\cdot\vec{\nabla}f$

vi) $\vec{\nabla}\times(f\vec{A}) = f(\vec{\nabla}\times\vec{A}) - \vec{A}\times(\vec{\nabla}f)$

## Second derivatives

i) $\vec{\nabla}\cdot(\vec{\nabla}T) = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \qquad (\nabla^2 T)$

ii) $\vec{\nabla}\times(\vec{\nabla}T) = 0$

iii) $\vec{\nabla}\cdot(\vec{\nabla}\times\vec{V}) = 0$

iv) $\vec{\nabla}\times(\vec{\nabla}\times\vec{V}) = \vec{\nabla}(\vec{\nabla}\cdot\vec{V}) - \nabla^2 V$

## Fundamental Theorems

$$\int_a^b (\vec{\nabla}T)\cdot d\vec{l} = T(b) - T(a)$$

↳ $\oint(\vec{\nabla}T)\cdot dl = 0$ (independent of path)

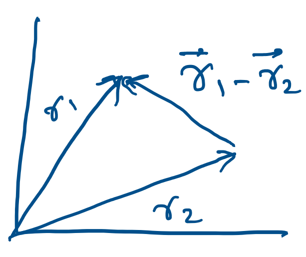
$$\int_V (\vec{\nabla}\cdot\vec{U})\,dV = \oint_S \vec{U}\cdot d\vec{a}$$ (total flow through a closed surface)

$$\int_S (\vec{\nabla}\times\vec{V})\cdot d\vec{a} = \oint \vec{V}\cdot d\vec{l}$$

## Coulomb Force

$$\vec{F}_{12} = \frac{K q_1 q_2}{r_{12}^2}\,\hat{r}_{12}$$

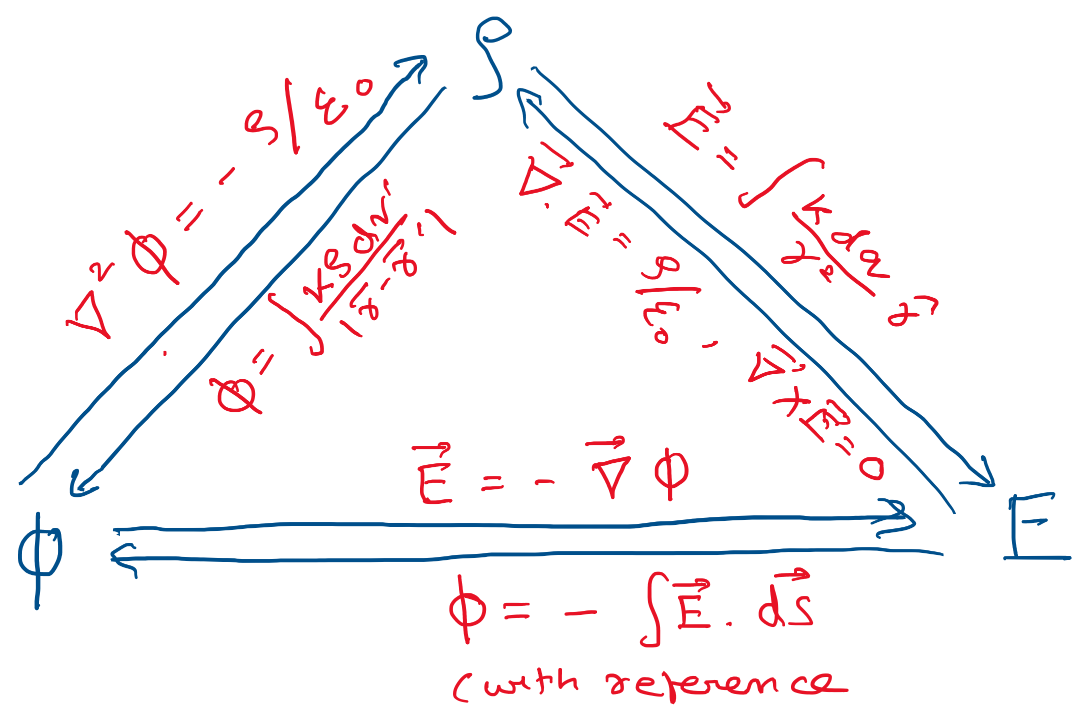
↳ on 1 due to 2



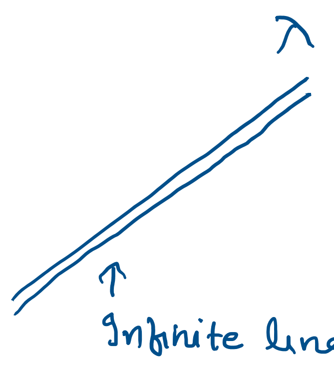

## Gauss's Law

$$\oint_S \vec{E}\cdot d\vec{a} = \frac{\int_V \rho\, dV}{\varepsilon_0}$$

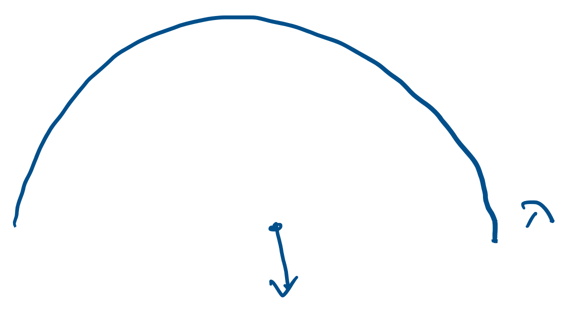
$$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$$




## Electric field & potential in various configurations

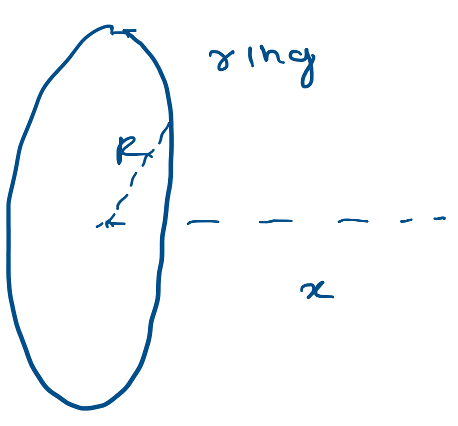



$$\vec{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{r}$$



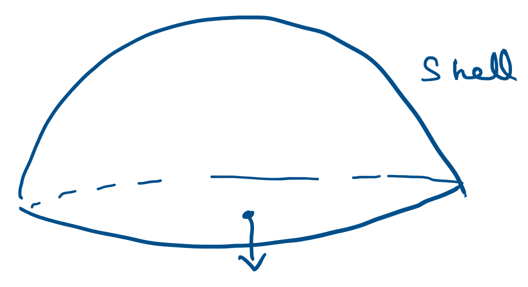

$$\vec{E} = -\frac{\lambda}{2\pi\varepsilon_0 r}\hat{z}$$

$$V = \frac{\lambda\pi R}{4\pi\varepsilon_0 R} = \lambda/4\varepsilon_0$$



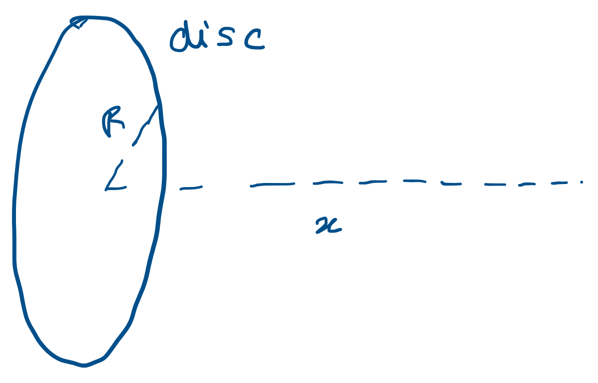

$$E = \frac{KQx}{(x^2+R^2)^{3/2}}\hat{x}$$

$$V = \frac{KQ}{(R^2+x^2)^{3/2}}$$



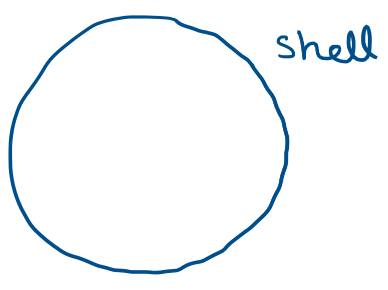

$$E = -K\pi\,\hat{z}$$

$$V = \frac{KQ}{R}$$




$$E_x = \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(R^2+x^2)^{1/2}}\right]\hat{x}$$

$$V = \frac{\sigma}{2\varepsilon_0}\left[(R^2+z^2)^{1/2} - z\right]$$



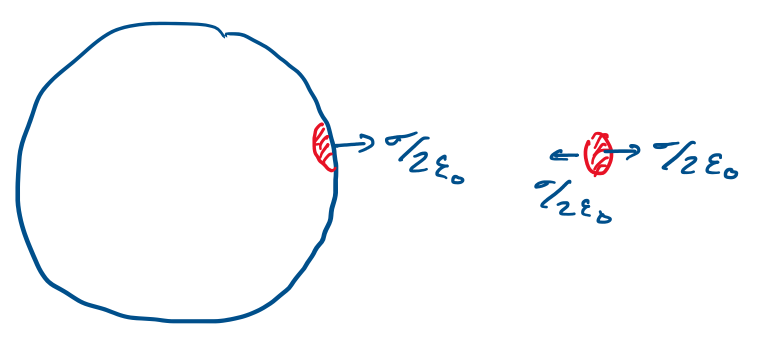

$E = KQ/r^2\,\hat{r} \qquad r > R$

$E = 0 \qquad r < R$

$V = KQ/r \qquad r > R$

$V = KQ/R \qquad r < R$

## Pressure due to $\vec{E}$ fields




Pressure $= \sigma^2/2\varepsilon_0$

Self energy of a charged shell $= \frac{KQ^2}{2R}$