

Lecture 9

Calculate the self inductance of a solenoid with (n -turns, l -length & ~~a~~ a -radius)

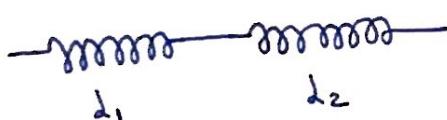
$$\Phi = (\mu_0 n I) \pi a^2 l$$

$$\therefore L = \mu_0 n^2 \pi a^2 l$$

If we now use this solenoid in a circuit with variable current, the voltage drop = $-L \frac{dI}{dt}$.

It is not necessary that the inductor is a solenoid. It can be any structure with some self inductance.

Using, the above, let us see how we can combine inductors in circuits.

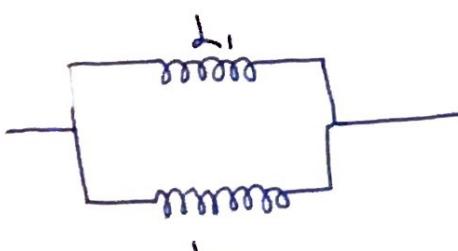


The voltage drop is the sum of voltage drop across individual inductors. But current is equal through both.

$$V_{\text{tot}} = V_1 + V_2$$

$$\therefore L \frac{dI}{dt} = L_1 \frac{dI}{dt} + L_2 \frac{dI}{dt}$$

$$\therefore L = L_1 + L_2$$



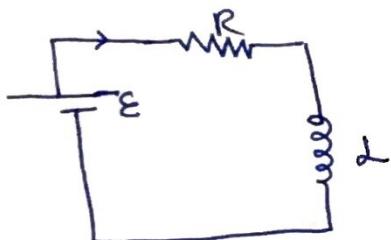
The current in each arm be I_1 & I_2 & voltage is same across both.

$$\therefore I = I_1 + I_2$$

$$\Rightarrow \cancel{L} \frac{dI}{dt} = \frac{dI_1}{dt} + \frac{dI_2}{dt}$$

$$\Rightarrow \frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2} \Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Let us now study a simple circuit.



At some arbitrary time t.

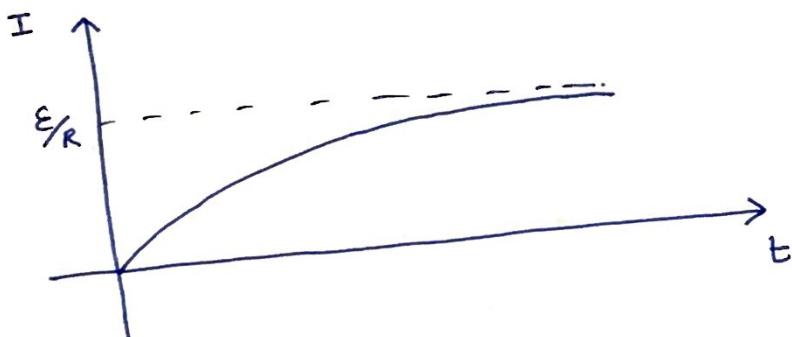
$$E - IR - L \frac{dI}{dt} = 0$$

$$\Rightarrow L \frac{dI}{dt} = E - IR$$

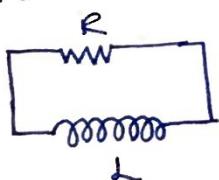
$$\Rightarrow \frac{dI}{E - IR} = \frac{dt}{L}$$

$$\Rightarrow I = \frac{E}{R} (1 - e^{-\frac{R}{L}t})$$

$$\therefore T = L/R$$



What happens if I now discharge the ~~excited?~~ inductor?



$$-L \frac{dI}{dt} - IR = 0$$

$$\Rightarrow -L \frac{dI}{dt} = IR$$

$$\Rightarrow I = I_0 e^{-Rt/L}$$

Using the information of the energy dissipated by the resistor, calculate the energy stored in the inductor.

$$U = \int_0^\infty RI^2 dt$$

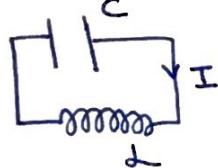
$$= \int_0^\infty RI_0^2 e^{-2Rt/L} dt = RI_0^2 \frac{L}{2R} = \frac{1}{2} L I_0^2$$

I wont solve it here as it has already been solved in the class

$$\text{Energy} = \frac{B^2}{2\mu_0}$$

Recall:- Similarly to the energy stored in electric fields, thus is the energy required to create the fields in the first place.

Circuit 2



Let us take a capacitor that is initially charged.

$$-L \frac{dI}{dt} + Q/C = 0$$

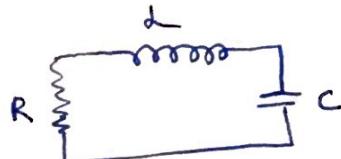
$$\Rightarrow L \frac{dI}{dt} = +Q/C$$

$$\Rightarrow \frac{d^2Q}{dt^2} = -\frac{Q}{LC}$$

$$\therefore Q = Q_0 \cos(\omega t + \phi)$$

$$\text{with } \omega = \frac{1}{\sqrt{LC}}$$

Circuit 3



Let us take the capacitor to be initially charged

$$-IR - L \frac{dI}{dt} + Q/C = 0$$

$$\Rightarrow L \frac{d^2Q}{dt^2} + R \frac{dQ}{dt} + Q/C = 0$$

$$\Rightarrow \frac{d^2Q}{dt^2} + \frac{R}{L} \frac{dQ}{dt} + \frac{Q}{LC} = 0$$

This requires a Q of the form

$$Q = Q_0 e^{-Rt/2L} \cos(\omega t + \phi)$$

$$\text{with } \omega = \sqrt{\omega_0^2 - \frac{R^2}{4L^2}}$$

Another helpful form of the same equation is

$$Q = e^{-Rt/2\omega} [A e^{i\omega t} + B e^{-i\omega t}]$$

We further define a quantity that helps ~~describ~~ describe the circuits.

$$Q \text{ (quality factor)} = 2\pi \times \text{no of cycles it takes for the energy (less the damping) to drop by } \frac{1}{e}$$

In our above example, energy stored is in the capacitor initially.

$$\therefore E_C \propto Q^2$$

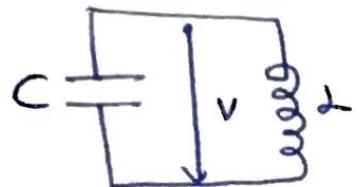
$$\therefore E_C \propto e^{-\frac{2RL}{\omega L}}$$

$$\therefore E_C \rightarrow \frac{1}{e} E_{C_0} \text{ at } t = \frac{L}{R}$$

$$\therefore \text{No of radians} = \omega t = \frac{\omega L}{R}$$

Alternatively Q may also be expressed in cycles and

then $Q = \frac{\omega L}{2\pi R}$



If set up the circuit such that initial conditions have voltage V_0 & varies as $V_0 \cos \omega t$. What is the energy stored at $t = 0$ & $t = \pi/2\omega$

For voltage $V_0 \cos \omega t$, the capacitor plate must hold

$$Q = C V_0 \cos(\omega t) \text{ charge.}$$

$$\therefore \text{Current through inductor} = -C V_0 \omega \sin(\omega t)$$

\therefore there is no energy in the inductor.

$$\therefore \text{Energy} = \frac{1}{2} C V_0^2 / 2$$

$$\text{At } t = \pi/2\omega$$

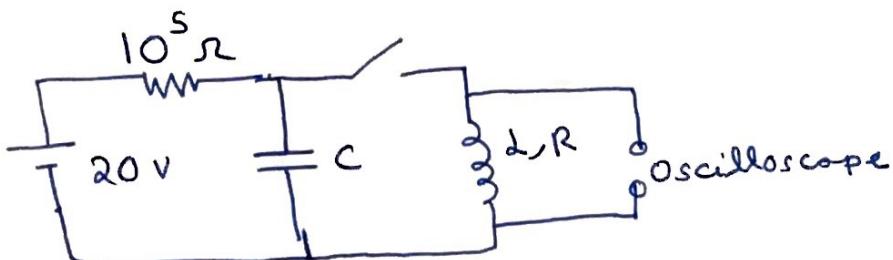
$$Q = 0$$

$$I = C V_0 \omega$$

$$\therefore \text{Energy} = \frac{1}{2} L C^2 V_0^2 \omega^2$$

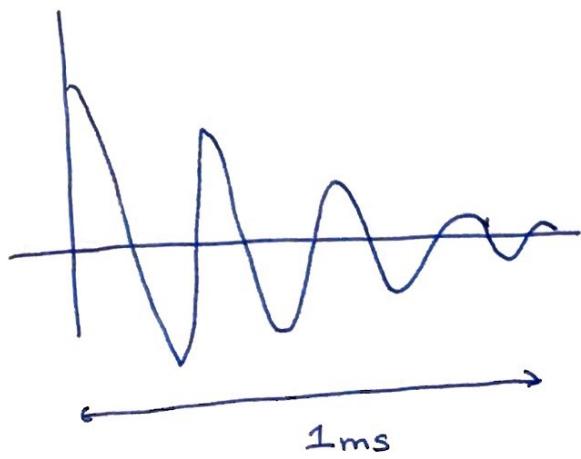
$$= \frac{1}{2} C V_0^2$$

Problem



$$L = 0.01 \text{ Henry}$$

Oscilloscope reading



$1/e$ in 0.5 ms

Consider the
10⁵ Ω large enough
that no current flows
through it.

- a) What is the 'C' in the circuit?
- b) Estimate 'R'.
- c) What is the voltage across the oscilloscope a long time after switch is opened?
- d) When the amplitude doesn't become negligible after a few cycles, you can consider $\omega \approx \frac{1}{\sqrt{LC}}$

Using $L = 0.01 \text{ H}$

$$\omega = 4 \times \frac{2\pi}{T} = 2.5 \times 10^4 \text{ s}^{-1}$$

$$\therefore C = 1.6 \times 10^{-7} \text{ F}$$

b) As we just studied, voltage goes to V_0 in $t = \frac{2L}{R}$

$$R = \frac{2L}{t}$$

$$= 40\Omega$$

c) After a long time, we essentially have two resistors,
 $10^5\Omega$ & 40Ω in series with $20V$.

$$\therefore V_{40\Omega} = \left(\frac{40}{10^5 + 40} \right) 20 = 0.008V$$