

Lecture 7

In the last class, we studied that the force experienced by a charge carrier at any point on a conductor is given by.

$$F = q (\vec{v} \times \vec{B}) \quad / \quad q \vec{E}$$

∴ the energy the charge gains as it goes through a loop is given by

$$\oint \vec{F} \cdot d\vec{l}$$

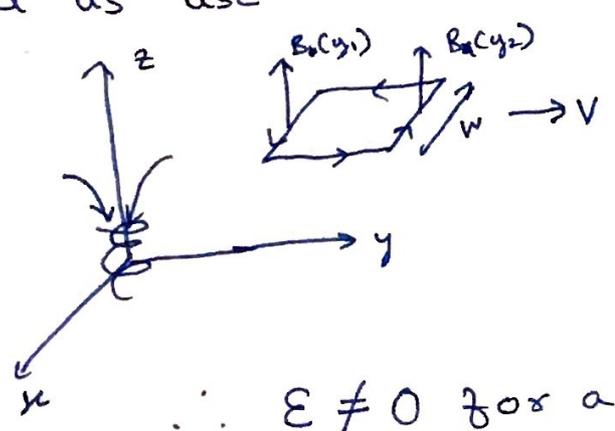
For a unit charge this turns out to be

$$\begin{aligned} \mathcal{E} &= \frac{\oint \vec{F} \cdot d\vec{l}}{q} \\ &= \frac{\oint (\vec{v} \times \vec{B}) \cdot d\vec{l}}{\oint \vec{E} \cdot d\vec{l}} \end{aligned}$$

(We should always consider the total force on a conductor. However, we generally tend to select frames where one or the other has no effect)

Here \mathcal{E} is the energy gained by a unit charge as it goes around a loop under the influence of field.

Let us use the above to find \mathcal{E}



$$\begin{aligned} \mathcal{E} &= \oint (\vec{v} \times \vec{B}) \cdot d\vec{l} \\ &= v B_0(y_1) w - v B_0(y_2) w \\ &= v w (B(y_1) - B(y_2)) \end{aligned}$$

$$\therefore \mathcal{E} = \frac{d\phi}{dt}$$

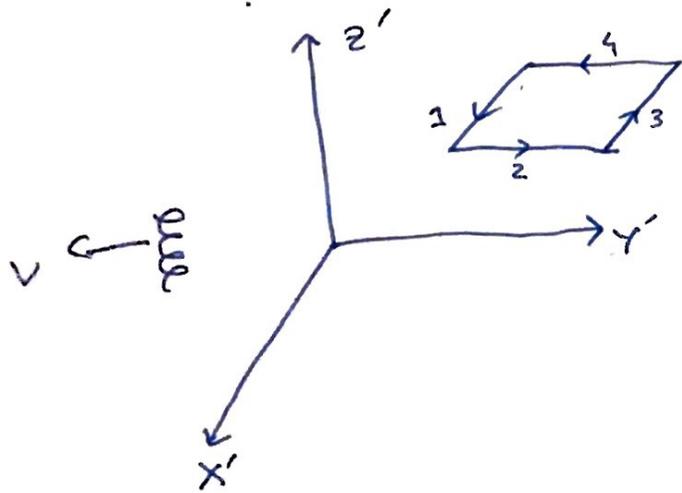
∴ $\mathcal{E} \neq 0$ for a loop.

∴ $\int \vec{f} \cdot d\vec{l}$ is a path dependent integral and we are dealing with non conservative fields (e.g. nonelectrostatic).

In nonconservative fields we no longer have the concept of ~~magnetic~~ potential but we shall still speak about voltage difference in circuits.

The watchword is that whenever you encounter \vec{B} (especially if the flux changes) Kirchoff's law is best abandoned. Proceed carefully!

What about the same system in the frame of the conductor?



We have no magnetic force acting on the loop as it is stationary.

$$\begin{aligned} \therefore \mathcal{E}' &= \int \vec{E}' \cdot d\vec{l}' \quad (E' = v \times B') \\ &= \int v B'(y_1') w - v B'(y_2') w \\ &= v w (B'(y_1') - B'(y_2')) \end{aligned}$$

$$\therefore \mathcal{E}' = \frac{d\phi'}{dt'}$$

$\therefore \mathcal{E} = \frac{d\phi}{dt}$ ~~also~~ holds in all frames but may not have the same value.

Let us stretch this result a bit further.

(I am dropping the ')

$$E = \int_C \vec{E} \cdot d\vec{s} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a}$$

$$\Rightarrow \int_S \vec{\nabla} \times \vec{E} \cdot d\vec{a} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{a}$$

For a sufficiently small 'S' bounded by a stationary 'C'

$$\vec{\nabla} \times \vec{E} = -\frac{d}{dt} \vec{B}$$

However as \vec{B} depends on both (x, y, z) & t we shall write

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

∴ to Review,

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \rho/\epsilon_0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(these are enough to give us the Maxwell equations in their final form but we will keep with the class)

Now let us compare

$$\vec{\nabla} \times \vec{B} = \frac{\mu_0 \vec{I}}{A} \Rightarrow$$

$$= \frac{\mu_0}{A} \frac{dq}{dt}$$



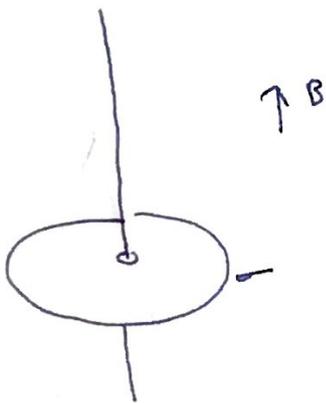
\vec{B} in $\hat{\theta}$ direction

We have

$$\vec{\nabla} \times \vec{E} = - \frac{\partial B}{\partial t} \Rightarrow \begin{array}{c} \uparrow \partial B / \partial t \\ \leftarrow \text{---} \rightarrow \\ \leftarrow \text{---} \rightarrow \end{array} \quad \vec{E} \text{ in } -\hat{\theta} \text{ direction}$$

Problem 1 (Teach last)

Consider a disc of radius a in a region of magnetic field \vec{B} acting upwards. The disc consists of stationary charges with surface charge density σ . The magnetic field is slowly turned off. Find final angular momentum.

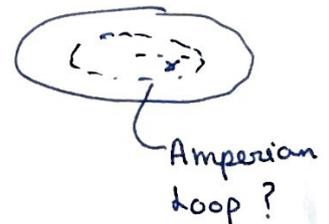


We know E is in θ direction from our discussion above and radially symmetric

$$\therefore \oint \vec{E} \cdot d\vec{l} = - \frac{d\phi}{dt}$$

$$\Rightarrow E_{\theta} 2\pi r = - \pi r^2 \frac{dB}{dt}$$

$$\Rightarrow E_{\theta} = - \frac{\pi r}{2} \frac{dB}{dt}$$



$$\begin{aligned} \therefore \text{Torque for this ring} &= \vec{r} \times \vec{F} \\ &= r \left(\sigma 2\pi r dr \left(-\frac{\pi r}{2} \right) \frac{dB}{dt} \right) \end{aligned}$$

$$\begin{aligned} \therefore \text{Total } \underline{\text{Momentum for ring}} &= \int \tau_{\text{ring}} dt \\ &= -\pi^2 r^3 dr \sigma \bullet (0 - B) \\ &= \pi^2 r^3 dr \sigma B \end{aligned}$$

$$\therefore \vec{L} \text{ Over whole disc} = \frac{\sigma B \pi^2 a^4}{4}$$

Energy Conservation? Angular Momentum Conservation?

Problem 2

A long solenoid carries a current $I(t)$. Find \vec{E} inside & outside solenoid in quasi static approximation

$$\begin{aligned}\vec{B} &= \mu_0 n I(t) \hat{z} & r < a \\ &= 0 & r > a\end{aligned}$$

∴ Inside: for Amperian loop of $r < a$

$$\Phi = B \pi r^2$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{d\Phi}{dt}$$

$$\Rightarrow E \times 2\pi r = -\mu_0 n \pi r^2 \frac{dI}{dt}$$

$$\Rightarrow E = -\frac{\mu_0 n r}{2} \frac{dI}{dt} \hat{\theta}$$

[Analogize ampere's law for thick wire]

∴ Outside: for amperian loop of $r > a$

$$\Phi = B \pi a^2$$

$$\Rightarrow E = -\frac{\mu_0 n \pi a^2}{2\pi r} \frac{dI}{dt} \hat{\theta}$$

$$= -\frac{\mu_0 n a^2}{2r} \frac{dI}{dt} \hat{\theta}$$