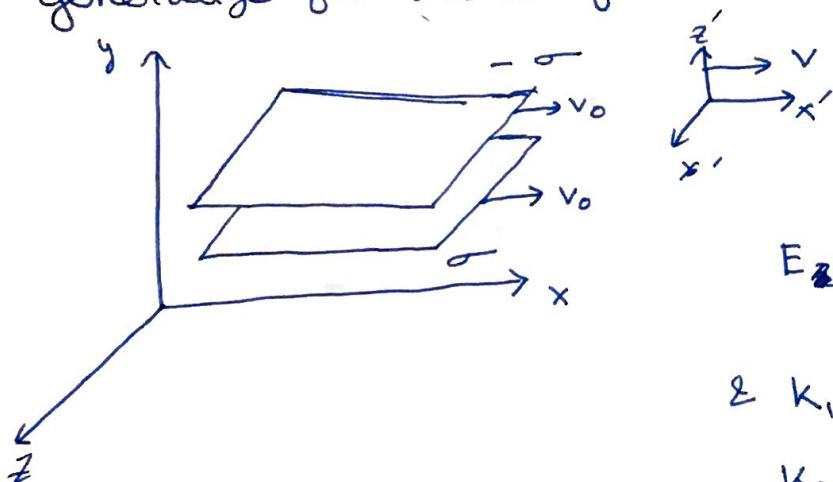


Lecture 5

Some of you had enquired about why we study \vec{A} at all. To be completely honest, while we shall see some mathematical advantages of using \vec{A} during the course, we shall mostly overlook its usage in engineering or physics. Instead, I will shortly hint on its application on astronomical phenomena towards the end of the class when I show you a video.

In the last class, we studied transformation of $E_{||}$ & E_{\perp} when moving from one frame to another. We also solved for $B_{||}$ when we studied the case of the solenoid.

However, our solution for E_{\perp} only applies for the special case when charges are at rest in one frame. We shall generalize for this \perp field today.



$$E_{xy} = \frac{\sigma}{\epsilon_0} (+g)$$

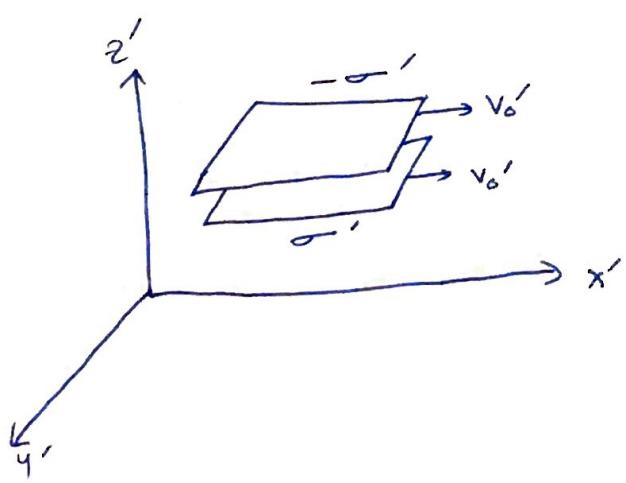
$$\& k_1 = -v_0 \hat{x} \text{ (for bottom sheet)}$$

$$k_2 = -v_0 (-\hat{x}) \text{ (for top sheet)}$$

$$B_{z3} = \mu_0 K (\hat{z})$$

$$v_{0'} = v_0 - v / (1 - v_0 v / c^2)$$

$$= c (\beta_0 - \beta) / (1 - \beta_0 \beta)$$



$$\gamma_0' = \gamma_0 \gamma (1 - \beta_0 \beta)$$

Now,

$$\sigma' = \sigma_{\text{rest}} \gamma_0'$$

$$= \frac{\sigma}{\gamma_0} \gamma_0'$$

$$\therefore E_y' = \frac{\sigma'}{\epsilon_0} = \frac{\sigma}{\epsilon_0} \frac{\gamma_0'}{\gamma_0}$$

$$= \frac{\sigma}{\epsilon_0} \gamma (1 - \beta_0 \beta)$$

$$= \gamma \left(\underbrace{\frac{\sigma}{\epsilon_0}}_{E_y} - v \frac{\cancel{\sigma} \mu_0 v_0}{\mu_0 \epsilon_0 c^2} \right) \quad \left(\& \mu_0 \epsilon_0 = \frac{1}{c^2} \right)$$

$$= \gamma (E_y - v B_z)$$

Similarly,

$$B_z' = \mu_0 k' = \mu_0 \sigma' v_0'$$

$$= \mu_0 \cancel{\frac{\sigma}{\epsilon_0}} \gamma \frac{(1 - \beta_0 \beta)}{(1 - \beta_0 \beta)} \cancel{c} \frac{(\beta_0 - \beta)}{(1 - \beta_0 \beta)}$$

$$= \gamma \left(\frac{v_0 \cancel{\mu_0 \sigma}}{c} - \frac{v}{c} \frac{\cancel{\mu_0 \sigma} c}{\cancel{\epsilon_0}} \right)$$

$$= \gamma \left(\underbrace{\frac{v_0 \mu_0 \sigma}{B_z}}_{B_z} - \cancel{\frac{\mu_0 \epsilon_0 \sigma}{E_y}} \right)$$

$$= \gamma (B_z - \frac{v}{c^2} E_y)$$

If we employ the same procedure for E_z & B_y , we get

$$E_z' = \gamma (E_z + v B_y) \quad \& \quad B_y' = \gamma (B_y + \left(\frac{v}{c^2} \right) E_z)$$

Summarizing,

$$\begin{aligned} E_x' &= E_x, \quad E_y' = \gamma (E_y - v B_z) \quad E'_z = \gamma (E_z + v B_y) \\ B_x' &= B_x \quad B_y' = \gamma (B_y + \frac{v}{c^2} E_z) \quad B'_z = \gamma (B_z - \frac{v}{c^2} E_y) \end{aligned}$$

or

$$\begin{aligned} E'_{||} &= E_{||} \quad \vec{E}'_{\perp} = \gamma (\vec{E}_{\perp} + \vec{v} \times \vec{B}_{\perp}) \\ B'_{||} &= B_{||} \quad \vec{B}'_{\perp} = \gamma (\vec{B}_{\perp} - (\frac{v}{c^2}) \times \vec{E}_{\perp}) \end{aligned}$$

The above formulation is in SI units. While Gaussian units make for more symmetrical looking equations, we will avoid them for the convenience of communication.

Special Cases

→ In frames where $B_{||} = 0$ & $B_{\perp} = 0$

$$\begin{aligned} B'_{||} &= 0 \quad \& \quad E'_{\perp} = \gamma E_{\perp} \\ \& \& \quad 2 B'_{\perp} = \gamma \left(-\frac{\vec{v}}{c^2} \times \vec{E}_{\perp} \right) \\ & & \quad = -\frac{\vec{v}}{c^2} \times E'_{\perp} \end{aligned}$$

$$\text{as } B'_{||} = 0 \quad \& \quad \vec{v} \times \vec{E}_{||} = 0$$

$$B' = -\left(\frac{\vec{v}}{c^2}\right) \times \vec{E}'$$

→ In frames where $E_{\perp} = 0$ & $E_{||} = 0$

$$E'_{\perp} = \gamma (\vec{v} \times \vec{B}_{\perp}), \quad E'_{||} = 0$$

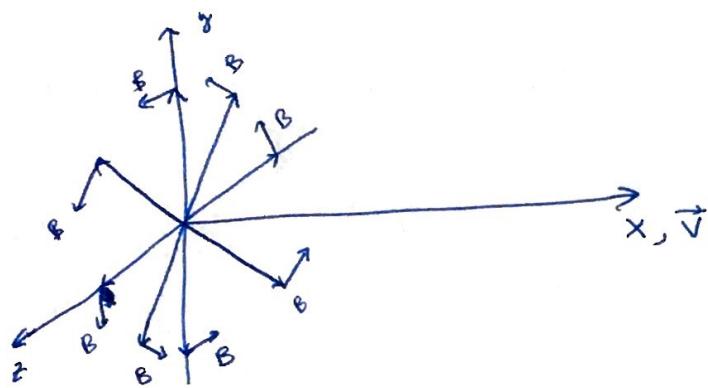
$$B'_{||} = B_{||}, \quad B'_{\perp} = \gamma \vec{B}_{\perp}$$

$$\therefore E' = \vec{v} \times B'$$

Using the above results, for the special case of a particle in motion, at near light speeds, the \vec{E}' is concentrated on a disc \perp to the direction of motion

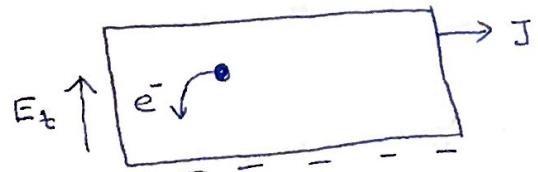
$$\therefore \text{using } \vec{B}' = \left(-\frac{\vec{v}}{c^2}\right) \times \vec{E}$$

\vec{B}' is \perp to both \vec{E}' & \vec{v}



I wont cover Rowland's experiment but I suggest you read in Purcell 6.8 as it is a lesson in the efforts that scientists have taken to unravel the mysteries of EM.

Hall Effect (I shall only discuss the force on a conductor here)



$$E_H = -|\vec{v} \times \vec{B}|$$

$$= \frac{|\vec{J} \times \vec{B}|}{(nq)}$$

Relativistic Capacitor

++ + + + + + + + + -
 $v_0 \rightarrow v_0$
 - - - - - - - - - -

(Ignore E & B by ~~charge~~ ~~capacitors~~)

a) What is the \vec{E} & \vec{B} by the capacitors in rest frame?

b) What is the \vec{E} & \vec{B} ~~in~~ the frame of the charge?

Use Lorentz Transformation & First Principle approaches
 to verify your answer.

a) $|\vec{E}| = \frac{\sigma}{\epsilon_0}$ (in \perp direction to motion of charge)

$$|\vec{B}| = 0$$

b) $E'_{||} = 0$

$$E'_{\perp} = \gamma (\epsilon_{\perp}) (-\hat{z}) = \frac{\gamma \sigma}{\epsilon_0} (-\hat{z})$$

$$B'_{||} = 0$$

$$B'_{\perp} = - \frac{\nabla}{c^2} \times E'_{\perp}$$

$$= - \frac{v_0}{c^2} \gamma \frac{\sigma}{\epsilon_0} (\hat{y})$$

Using first principle

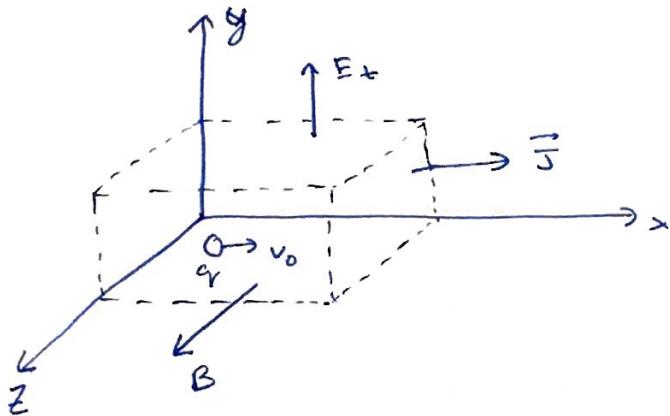
$$\sigma' = \gamma \sigma$$

$$E' = \gamma \frac{\sigma}{\epsilon_0} (-\hat{z})$$

$$B' K' = - \sigma' v_0 \hat{x} = - \gamma \sigma v_0 \hat{x}$$

$$\begin{aligned}\therefore \vec{B} &= \mu_0 \vec{k} I (\hat{y}) \\ &= \mu_0 r o - v_0 \hat{y} \\ &= \frac{v_0}{c^2} r \frac{o}{\epsilon_0} \hat{y}\end{aligned}$$

Hall effect in Semiconductors



a) What is the force on the particle?

b) Let n be the number of charged particles per unit volume travelling with the current. What is the force per unit volume?

For what E_t does net force become zero.

c) For n-type semiconductors, we get $J > 0, B > 0$ & $E_t > 0$.

What are the charge carriers?

For p-type semi-conductors, we get $J > 0, B > 0$ & $E_t > 0$.

What are the charge carriers?

a) $\vec{F} = q [\vec{E} + \vec{v} \times \vec{B}] = q [E_t - v_0 B] \hat{y}$

b) $\vec{F} = nq [E_t - v_0 B] \hat{y}$ & $E_t = J B / nq$

c) $J > 0, B > 0, E_t < 0 \Rightarrow q < 0$

$J > 0, B > 0, E_t > 0 \Rightarrow q > 0$