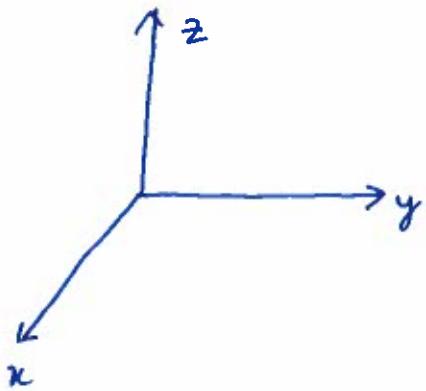
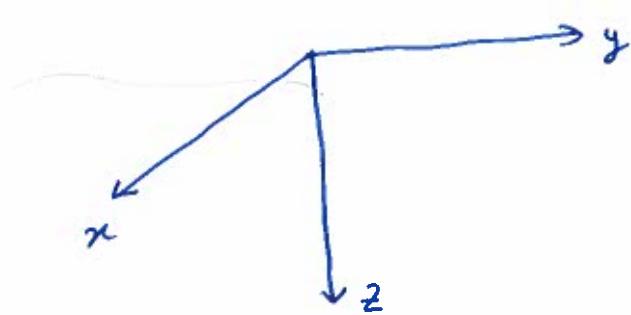


* Note on right handedness of our ~~Cartesian~~ coordinate systems.



Right-handed
System

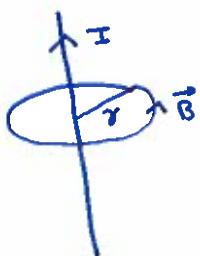


Left-handed
system.

. . . to find cross product of two vectors, right hand wrap rule is an efficient and universal tool.

Lecture 2

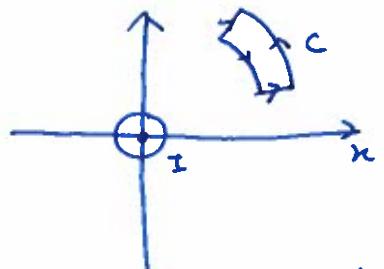
(This class is theory heavy)



$$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$$

Experimentally ascertained.

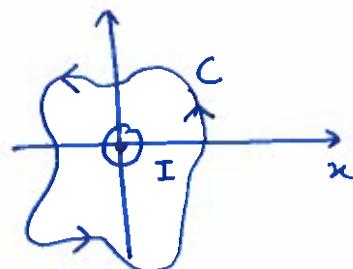
Pave showed in class



$$\oint_C \vec{B} \cdot d\vec{l} = 0$$

when the curve
doesn't enclose
a line current

This can be extended for any
arbitrary curve composed of curve
of the type shown above.



$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

where $\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$
 \uparrow or Tm/A
Permeability of free space

We usually use simple 'C's over which \vec{B} doesn't vary to make life easy.

Note:- Our equations in general here assume time independent current. i.e. charges in motion with constant velocity. Accelerating charges have a whole different physics that we shall be studying soon. However, assume for n. Only for Ampere's law we shall ~~assume~~ that they hold for bent wires too.

Differential form

$$\oint_C \vec{B} \cdot d\vec{l} = \mu_0 I$$

$$\Rightarrow \oint_C \vec{B} \cdot d\vec{l} = \mu_0 \int_S \vec{J} \cdot d\vec{a}$$

where S is the surface enclosed by curve

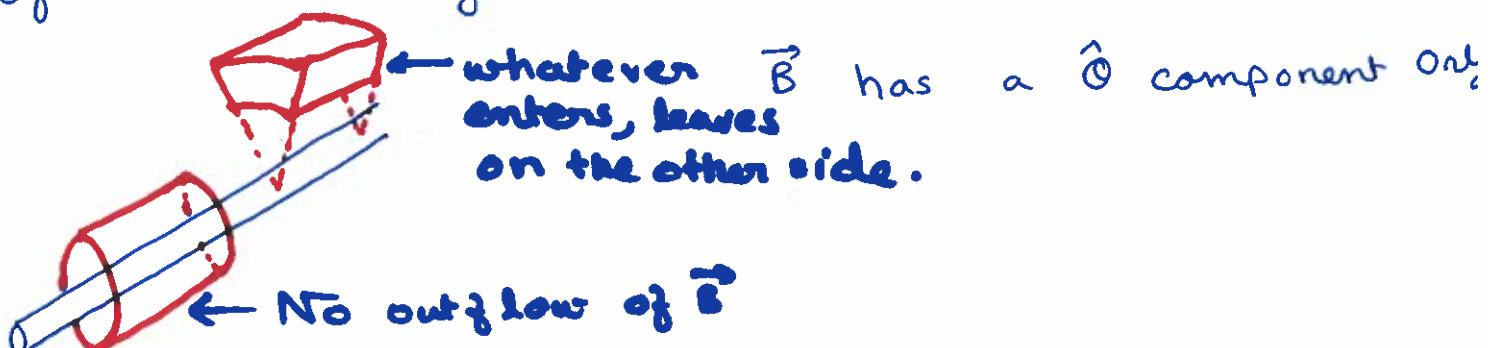
$$\text{Using Stokes law, } \oint_C \vec{F} \cdot d\vec{l} = \int_S (\vec{\nabla} \times \vec{F}) \cdot d\vec{a}$$

$$\Rightarrow \int_S (\vec{\nabla} \times \vec{B}) \cdot d\vec{a} = \int_S \mu_0 \vec{J} \cdot d\vec{a}$$

$\therefore \vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$

Qualitative Description of divergence of \vec{B}

We discuss for a straight line current. All other configurations can be considered to be superposition of several straight line currents.



$$\therefore \vec{\nabla} \cdot \vec{B} = 0$$

Theorem 6.1 in Purcell & Morin discusses the uniqueness of \vec{B} when $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$ and $\vec{\nabla} \cdot \vec{B} = 0$.

To Review

Electrostatics

$$\vec{\nabla} \cdot \vec{E} = \sigma / \epsilon_0$$

$$\vec{\nabla} \times \vec{E} = 0$$

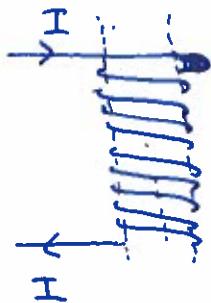
Magnetostatics

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{B} = \mu_0 J$$

Problem 1

Find the magnetic field of a very long solenoid consisting of ∞ closely wound ~~say~~ turns of Radius 'R' and carrying a steady current I.

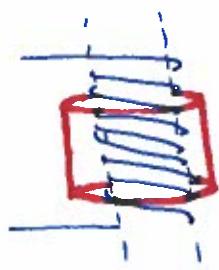


We consider the wires to be nearly horizontal.

Solution

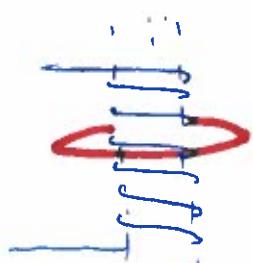
Using some hindsight, we shall use the cylindrical coordinate system.

$B_z \rightarrow$ must be zero otherwise $\nabla \cdot \vec{B}$ won't be zero



The dot product of the magnetic field \perp to the top and bottom surfaces with da must cancel as they the magnetic field both at the upper and lower surface must be in the same direction.

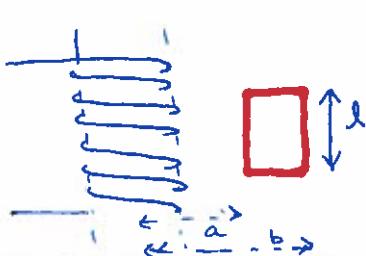
$B_\theta \rightarrow$



I_{enc} by amperian loop is zero.

$$\therefore B_\theta = 0$$

$B_z \rightarrow$



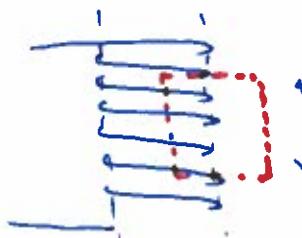
Let us consider $B_z(z)$

$$\oint B_z dz \cdot dl = \mu_0 I_{enc}$$

$$\Rightarrow B_z(b) - B_z(a) = 0$$

Now for $b = \infty$, $B_z(\infty)$ must be zero for \vec{B} to be physical.

$$\therefore B_z(a) = 0 \text{ for arbitrary } a, a > R.$$



$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enc}}$$

$$\Rightarrow B_z(z) l = \mu_0 n i l$$

$$\Rightarrow B_z(z) = \mu_0 n i$$