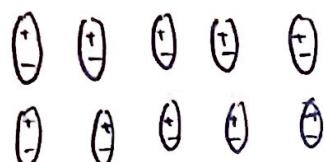


Lecture 15

In the last class we discussed about dipoles and the polarizability of atoms. Now let's consider a large number of molecules with some dipole moment \vec{P} and density N . We shall ignore if this moment is permanent or induced for now.

So, in bulk let us assume that these dipoles point in the same direction

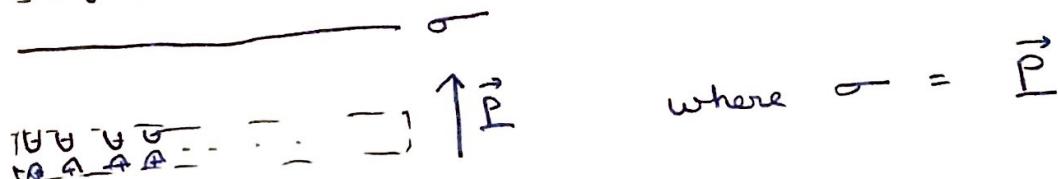


The polarization density for this configuration is given by $\vec{P} = \vec{p}N$.

∴ for a small volume dV , the net dipole moment is $\vec{P}N$

This is a reasonable approximation when N is very large and we use it to calculate for ϕ / E outside the volume containing the dipoles. i.e. the dipoles are very small considering the length scales involved.

Using these considerations, we found in the class that, for a block of such molecules having polarization density \vec{P} , we can treat the bulk as being neutral and all charges are accumulated at the surface.



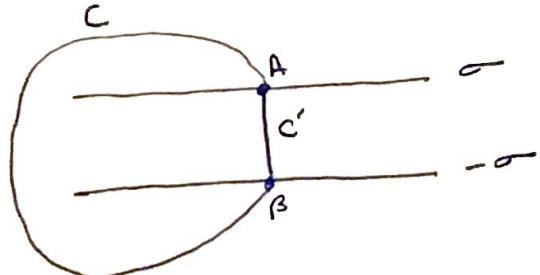
$$\text{where } \sigma = \frac{\vec{P}}{V}$$



But this doesn't tell us enough about the field inside the bulk.

We can indeed speak about the field outside.

This is because close to the molecules, fields can be in the scale of several million V/m. Instead we use the average field.



$$\Delta V_C = \Delta V_{C'} \quad (\text{electrostatic fields})$$

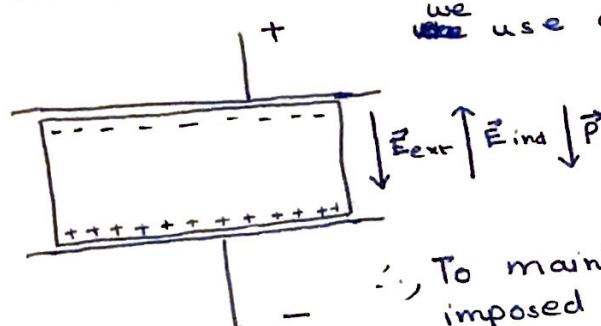
as the potential difference ΔV_C can be simply created by an \vec{E} pointing downward in the bulk, we say that

$$\langle E_{in} \rangle = \frac{\sigma}{\epsilon_0} \text{ (downward)} = -\frac{\vec{P}}{\epsilon_0}$$

Moving forward we shall simply refer to this as E_{in} and it should be understood as the average.

Now the question remains on how we could characterize the material involved.

To do this, we use ~~an~~ capacitors,



\therefore To maintain the voltage difference imposed by the battery, we need more charge on the capacitor plates.

Let this new charge be Q_K .
and Q' be the charge on the dielectric surface

$$\therefore \frac{Q_K}{A\epsilon_0} - \frac{Q'}{A\epsilon_0} = \frac{Q}{A\epsilon_0} \quad (\text{by equating voltage before and after inserting capacitor})$$

$$\Rightarrow \vec{E}_K - \frac{\vec{P}}{\epsilon_0} = \vec{E} \quad (\text{This 'K' is known as the dielectric constant. The higher the value, the more dipole moment the material produces})$$

$$\Rightarrow \vec{P} = \vec{E} (K-1) \epsilon_0$$

$\xrightarrow{x_e \rightarrow \text{electric susceptibility}}$

$$(K_{vac} = 1)$$

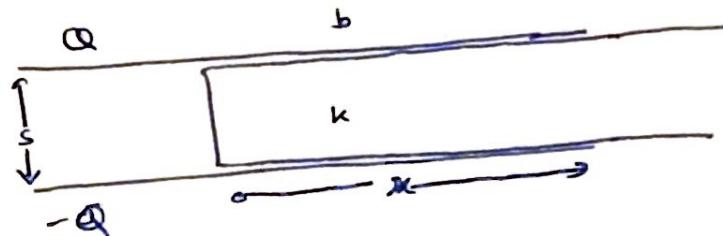
Further, materials where $\vec{P} \propto \vec{E}$ are known as linear dielectrics.
This is the simplest of scenarios as x_e may be dependent on \vec{E} itself or x_e may be a non isotropic material.

We shall only focus on homogenous & isotropic materials alone.

$$\text{For the above scenario } C' = \frac{Q'}{V} = \frac{Q \cdot K}{\frac{Q}{A\epsilon_0} d} = \frac{A\epsilon_0 K}{d} = CK$$

Problem 1

A rectangular capacitor with side lengths $a \times b$ has separation ' s ' with ' s ' much smaller than ' a ' and ' b '. It is partially filled with a dielectric with constant K . The overlap distance is x . The capacitor is isolated and has constant charge Q .



a) Find energy stored in the system?

b) Find the force on the dielectric.

c) We treat this as two parallel capacitors.

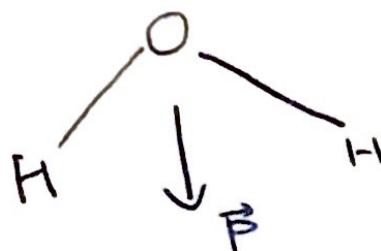
$$\begin{aligned} C_{\text{tot}} &= C_1 + C_2 \\ &= \frac{\epsilon_0 a (b-x)}{s} + \frac{\epsilon_0 a x K}{s} = \frac{\epsilon_0 a}{s} [b + (K-1)x] \end{aligned}$$

$$\therefore U = \frac{Q^2}{2C} = \frac{Q^2 s}{2\epsilon_0 a [b + (K-1)x]}$$

$$b) F = -\frac{dU}{dx} = \frac{Q^2 s (K-1)}{2\epsilon_0 a [b + (K-1)x]}$$

Problem 2

The electric dipole moment of water molecule is $6.13 \times 10^{-30} \text{ C-m}$. Now imagine you could make all water molecules point down. Calculate the resultant surface charge density. Find the no of extra $\bar{\sigma}$'s on the surface



$$N = \frac{6 \times 10^{23}}{18 \text{ cc}} \quad \begin{matrix} \leftarrow \text{No. of molecules} \\ \leftarrow \text{volume of mol} \end{matrix}$$

$$= 3.33 \times 10^{22} \text{ cm}^{-3}$$

Now $\vec{P} = \vec{P}^N$

$$= (6.13 \times 10^{-30} \text{ cm}) \times (3.33 \times 10^{22} \text{ cm}^{-3})$$

$$= 2.04 \times 10^{-5} \text{ C/cm}^2$$

$$\sigma = \vec{P}$$

$$\therefore n_e = \frac{\sigma}{e} = 1.3 \times 10^{-19} \text{ cm}^{-2}$$