

## Lecture 11

Q) Why even study the  $e^{i\omega t}$  method if it can only solve for sinusoidally varying sources?

→ All sources can be Fourier expanded in terms of sine functions.

\* Kirchhoff's Law in Circuit.

— Just like voltage law, conservation of charge must also hold. ∴ at all junction  $\sum I_i(t) = 0$

Note :- Our whole analysis works for linear circuit elements alone i.e. for elements with  $V \propto I$ . Fortunately, RLC are all linear.

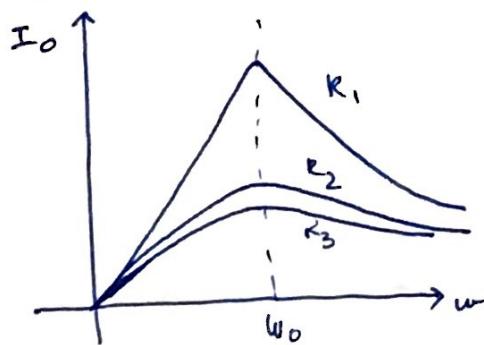
## Series RLC Circuits

From last class,

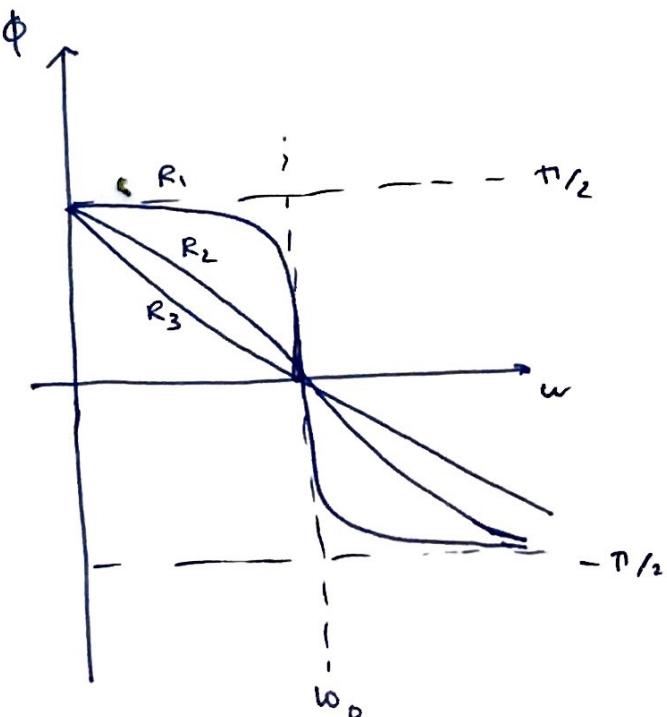
$$I_0 = \frac{E_0 e^{i(\omega t - \phi)}}{\sqrt{R^2 + (w_L - w_{LC})^2}}$$

where  $\phi = \tan^{-1} \left( \frac{w_L - w_{LC}}{R} \right)$

Plotting for these



$$R_1 < R_2 < R_3$$



An useful quantity while discussing these circuits, in the frequency range in which power becomes half.

For half power

$$I_0 = \frac{I_{\max}}{\sqrt{2}}$$

This is possible for  $(\omega_L - \frac{1}{\omega_C}) = R$

Now,

$$\omega_L - \frac{1}{\omega_C} = \omega_0 L \left( 1 + \frac{\Delta\omega}{\omega_0} \right) - \frac{1}{\omega_0 C \left( 1 + \frac{\Delta\omega}{\omega_0} \right)}$$

$$= \omega_0 L \left( 1 + \frac{\Delta\omega}{\omega_0} - \frac{1}{1 + \Delta\omega/\omega_0} \right)$$

$$= \omega_0 L \left( \frac{2\Delta\omega}{\omega_0} \right)$$

$$\Rightarrow \left( \frac{2\Delta\omega}{\omega_0} \right) \omega_0 L = R$$

$$\Rightarrow \frac{2\Delta\omega}{\omega_0} = \frac{R}{\omega_0 L}$$

[g will just use this result]  
here  $Q = \omega_0 L / R$

$$\Rightarrow 2\Delta\omega = \omega_0 / Q$$

## Power for AC Circuits

$$P = VI$$

$$= E_0 \cos(\omega t) \frac{E_0}{Z} \cos(\omega t + \phi)$$

$$= \frac{E_0^2}{Z} (\cos^2(\omega t) \cos \phi - \cos(\omega t) \sin(\omega t) \cos \phi)$$

$$\bar{P} = \frac{E_0^2}{2Z} \cos \phi$$

Note  $\frac{E_0}{\sqrt{2}} = V_{rms}$ , &  $\frac{I_0}{\sqrt{2}} = I_{rms}$

$$\therefore \bar{P} = V_{rms} I_{rms} \cos \phi$$

\* Calculate the power dissipated for a series RLC circuit & interpret it.

$$\text{For series RLC} \quad \tan \phi = \frac{\omega L - 1/\omega C}{R}$$

$$\therefore \cos \phi = \frac{R}{Z}$$

$$\therefore \bar{P} = V_{rms} I_{rms} R/Z$$

∴ Only the resistor fraction of the circuit dissipates any energy.

## Power for AC Circuits

$$P = V I$$

$$= E_0 \cos(\omega t) \frac{E_0}{Z} \cos(\omega t + \phi)$$

$$= \frac{E_0^2}{Z} (\cos^2(\omega t) \cos \phi - \cos(\omega t) \sin(\omega t) \cos \phi)$$

$$\bar{P} = \frac{E_0^2}{2Z} \cos \phi$$

Now  $\frac{E_0}{\sqrt{2}} = V_{rms}$ , &  $\frac{I_0}{\sqrt{2}} = I_{rms}$

$$\therefore \bar{P} = V_{rms} I_{rms} \cos \phi$$

\* Calculate the power dissipated for a series RLC circuit & interpret it.

$$\text{For series RLC} \quad \tan \phi = \frac{\omega L - 1/\omega C}{R}$$

$$\therefore \cos \phi = \frac{R}{Z}$$

$$\therefore \bar{P} = V_{rms} I_{rms} R/Z$$

Only the resistor fraction of the circuit dissipates any energy.

\* Calculate the equivalent power factor for parallel RLC circuit.  
Start from calculating  $Z$ .

$$Z = \frac{1}{\frac{1}{R} + i\omega C + \frac{1}{i\omega L}}$$

$$= \frac{1}{\frac{1}{R} + i(\omega C - \frac{1}{\omega L})}$$

$$= \frac{\frac{1}{R} - i(\omega C - \frac{1}{\omega L})}{\left(\frac{1}{R}^2 + (\omega C - \frac{1}{\omega L})^2\right)}$$

$$= \underbrace{\frac{1}{\sqrt{(\gamma_R)^2 + (\omega C - \gamma_{\omega L})^2}}}_{Y_{Zmag}} \left( \frac{\gamma_R}{(\gamma_{Zmag})} - \frac{i(\omega C - \gamma_{\omega L})}{(\gamma_{Zmag})} \right)$$

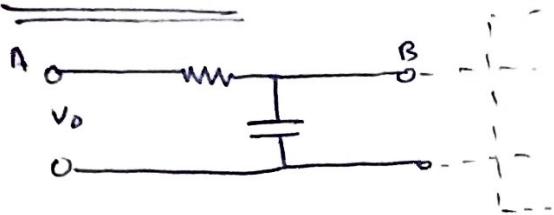
$$\therefore \cos \phi = \frac{\gamma_R}{\sqrt{(\gamma_R)^2 + (\omega C - \frac{1}{\omega L})^2}}$$

$$\therefore \bar{P} = I_{rms} V_{rms} \cos \phi$$

$$= \frac{1}{2} E_o \frac{E_o}{Z} \frac{\gamma_R}{\gamma_Z} = \frac{E_o^2}{2R}$$

Yet again only the resistance contributes to the ~~power~~ dissipation in circuit.

## Problem 2



An AC source of  $V_o \cos(\omega t)$  is applied across terminals at A. B is connected to a very high impedance.

- 1) Calculate  $|V_1/V_o|^2$
- 2) Choose R & C to make this ratio 0.1 for 5000 Hz signal. Interpret this.
- 3) Show that at high frequency, signal power is reduced by  $\gamma_4$  for every doubling of frequency.
- 4) How can we make it  $\gamma_{10}$  for every such doubling.

a)  $\tilde{V} = V_o e^{j\omega t}$

$$\tilde{I}_C = \tilde{V}/Z = \frac{\tilde{V}}{R + \frac{1}{j\omega C}} = \frac{\tilde{V}}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} e^{-j \tan^{-1}\left(\frac{1}{\omega C R}\right)}$$

$$\therefore \tilde{V}_C = \frac{\tilde{V}}{\sqrt{R^2 + (\gamma_{10} C)^2}} e^{-j \tan^{-1}(\gamma_{10} \omega C)}$$

$$= \frac{\tilde{V}}{\sqrt{1 + (R \omega C)^2}} e^{j(\tan^{-1}(\gamma_{10} \omega C) - \pi/2)}$$

$$\therefore |V/V_o|^2 = \frac{1}{1 + R^2 \omega^2 C^2}$$

b)  $\frac{1}{1 + \omega^2 R^2 C^2} = 0.1 \Rightarrow \omega^2 R^2 C^2 = 9 \Rightarrow R C = \frac{3}{\omega} = 10^{-4} s$

You can choose your combo of RC

⇒ This is a very basic version of low pass filter.

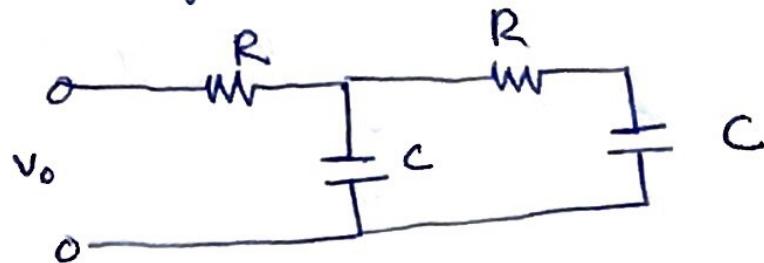
c)  $\left| \frac{V_1}{V_0} \right|^2 = \frac{1}{1 + \omega^2 R^2 C^2}$

for very high frequency

$$\left| \frac{V_1}{V_0} \right|^2 = \frac{1}{\omega^2 R^2 C^2}$$

Here every doubling of frequency reduces the <sup>power</sup> energy by a factor of 4.

d) We just add another loop.



$$\left| \frac{V_1}{V_0} \right| = \frac{1}{4}$$

$$\left| \frac{V_2}{V_1} \right| = \frac{1}{4}$$