

Lecture 1

"The longer your ignorance, the stronger the magnetic field"
— Woltjer 1966

Welcome to the final term of Ph 1. Here, you will finally begin to apply all the powerful tools you learnt last term and eventually study Maxwell's equation. The importance of this course cannot be overestimated as the theory of EM that you shall study is what every theory in physics of EM that you shall study is what every theory in physics aspires to be. We shall end the term with a study of radiation due to accelerating charges.

∴ You can divide the last two term as

- 1B → Physics of charges at rest (Electrostatics)
Physics of boosting to reference frames (SR)
- 1C → Physics of charges in motion
 - ↳ Constant Velocity
(immediate consequence of term 1B)
 - ↳ Acceleration.

TA Info

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Rec Sessions :- M/W 11:00AM at 155 ARMS
Quiz Review :- TBD.
(Class on 6th June will be taken by Rana Adhikari)

Text

Purcell & Morin's 'Electricity & Magnetism'.

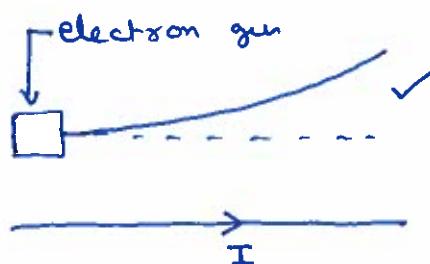
While historically, magnetism developed from the experiments of Faraday, Oersted, etc, the physical nature of magnetism is best realized when applying SR to charges at rest.

However, as we have students who may not have been introduced to magnetism before, we shall take a more convenient method where we shall follow chronology where it suites us and discard it where we need to obtain a ~~different~~ deeper insight.

Lorentz Force

Oersted was the first to notice that currents in wire (which were neutral) experienced a force between them. This was shown to you by Dave in class. Oersted further showed that currents in wire tend to deflect compass needles. ∴ currents in wires had some 'action at distance' component associated to it.

We shall instead deal with a slightly different setup.



∴ Charges feel an extra force in the presence of current carrying wires.

We write the force experienced as $\vec{F}_{\text{mag}} = q(\vec{v} \times \vec{B})$

(Experimentally obtained)

(Discuss hand sign)

Important Properties to Remember

1) Charge is invariant when moving from one frame to another

$$\left(\int_{S(t)} \vec{E} \cdot d\vec{a} = \int_{S'(t')} \vec{E}' \cdot d\vec{a}' \right)$$

2) Magnetic field does no work.

$$\vec{F}_{\text{mag}} = q(\vec{v} \times \vec{B})$$

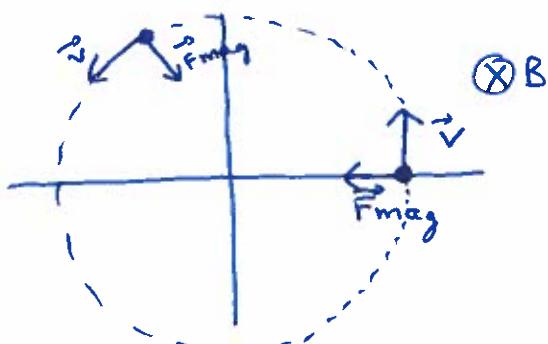
$$dW = \vec{F}_{\text{mag}} \cdot \vec{dl}$$

$$= q(\vec{v} \times \vec{B}) \cdot \frac{\vec{dl}}{dt} dt$$

$$= q(\vec{v} \times \vec{B}) \cdot \vec{v} dt$$

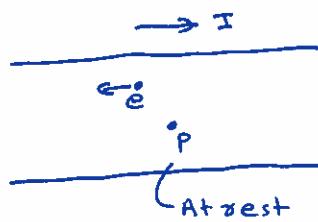
$$= 0$$

3) But it can accelerate.



Alternate form

For a current carrying wire,



Let the ~~electron~~ linear charge density for moving charges be λ

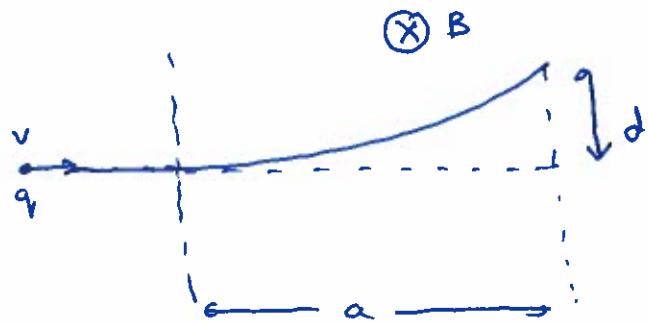
$$\therefore \vec{I} = \frac{d\vec{q}}{dt} = \lambda \frac{d\vec{l}}{dt} = \lambda \vec{v}$$

$$\therefore \vec{F}_{\text{mag}} = \int (\vec{v} \times \vec{B}) dq = \int (\vec{v} \times \vec{B}) \lambda dl = \int (\vec{I} \times \vec{B}) dl = I \int (\vec{dl} \times \vec{B})$$

Here I assumed that \vec{I} & $d\vec{l}$ are in the same direction and used the relation $T = \lambda \cdot l$ in the second term

Let us look at some problems now.

- I. A particle of charge q enters a region of uniform \vec{B} (x)



The field deflects a particle a distance d from the original line of flight.

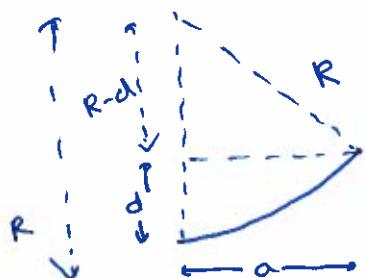
i) Is the charge positive or negative?

ii) Find the momentum of the particle.

i) Positive. We use $\vec{v} \times \vec{B}$ to obtain the direction of force on a positive charge and compare.

ii) Magnetic force is centripetal as it acts \perp to \vec{v} .

∴ let us consider a circle of radius R , where R is the radius of curvature of the path traversed by the particle.



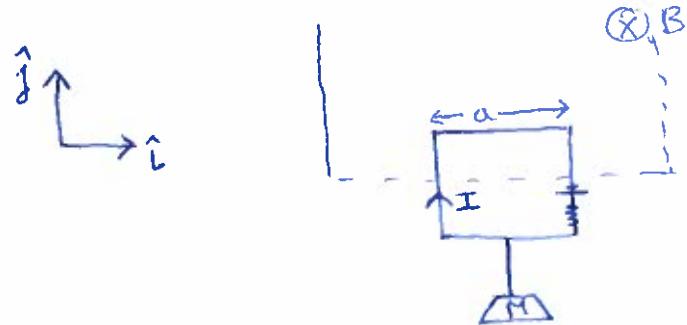
$$\begin{aligned} R^2 &= (R-d)^2 + a^2 \\ \Rightarrow R^2 &= R^2 - 2Rd + d^2 + a^2 \\ \Rightarrow R &= \frac{d^2 + a^2}{2d} \end{aligned}$$

$$\therefore \frac{mv^2}{R} = qVB \quad (\text{Centripetal force due to magnetic field})$$

$$\begin{aligned} \Rightarrow P &= qBR \\ &= qB \frac{(a^2 + d^2)}{2d} \end{aligned}$$

Problem 2

A rectangular loop of wire supporting a mass ' m ' hangs vertically with one end in uniform magnetic field $B \hat{j}$. i) For what current in the loop, would the magnetic force balance the gravitational force.



$$\vec{F}_{\text{mag}} = \int (\vec{dl} \times \vec{B}) \vec{I}$$

$$= I B a \hat{j}$$

$$\therefore I B a = mg \text{ (to balance)}$$

$$\Rightarrow I = \frac{mg}{Ba}$$

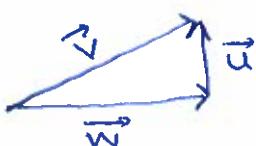
ii) What happens if g increase the current?

It rises.

iii) But didn't we just discuss that the magnetic field does no work?

Well, it still doesn't.

Let us look at a single point charge on the wire. Its motion can be decomposed into



where \vec{V} is the net velocity of the charge.

$$\text{Force due to } \vec{w} = q \vec{w} B (\hat{j})$$

$$\text{Force due to } \vec{u} = q \vec{u} B (-\hat{i})$$

\therefore The magnetic field tries to oppose charge from flowing in at a higher rate. Work needs to be done by the battery to even maintain a current higher than what is