Lecture 8

Sunday, 27 October 2019 6:53 PM

Definition :- $\vec{p} = m\vec{v}$

Fundamental law :- Momentum is always conserved.

Additionally, for larger bodies or multiparticle systems, it is often helpful to reduce the behaviour of the whole system to that of a particle. The location of such a particle is defined by

$$\vec{R} = \frac{\Sigma_i m_i \vec{r_i}}{\Sigma m_i}$$

Center of mass position

Differentiating

$$\vec{V} = \frac{\Sigma_i m_i \vec{v_i}}{M} \Rightarrow$$ Momentum of center of mass is sum of all momentum.

&

$$\vec{A} = \frac{\Sigma_i m_i \vec{a_i}}{M} \Rightarrow$$ Force on center of mass is equal to sum of all external forces. Any internal forces cancel out.

Lets dive right into a few problems.

1) A snowball is thrown against a wall. Where does its momentum go? Where does its energy go?

→ The momentum is transferred to earth and makes it revolve and rotate a bit faster.

$$M(\Delta V) \simeq mv$$

$$\therefore \Delta V \simeq \frac{m}{M} v$$

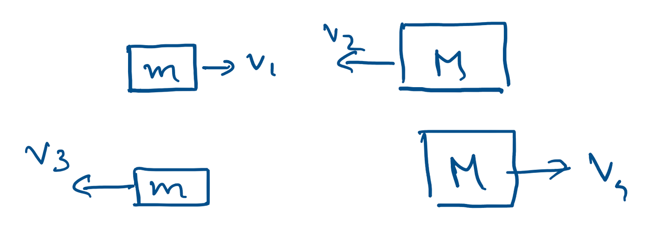
In the frame where the earth was at rest, its energy is

$$\frac{1}{2} M (\Delta V)^2 = \frac{1}{2} m v^2 \left(\frac{m}{M}\right) << \frac{1}{2} m v^2$$

∴ the earth takes up all of the momentum but pretty much none of the energy ∴ energy converts to heat and melts the snow.

2) <u>1 D Collissions</u>

Mass 'm' moves with velocity v_1 to the right and Mass 'M' move with velocity v_2 to the left. After an elastic collission find v_3 of 'm' and v_4 of 'M'



Conservation of Momentum

$$mv_1 - Mv_2 = -mv_3 + MV_4$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} M v_2^2 = \frac{1}{2} m v_3^2 + \frac{1}{2} M v_4^2$$

Solving them, we get

$$v_3 = \frac{(M-m)v_1 + 2Mv_2}{M+m}$$

$$v_4 = \frac{2mv_1 - (M-m)v_2}{M+m}$$

The above requires you to solve a quadratic equation

Insted, lets employ a trick,

$$m(v_1 + v_3) = M(V_2 + V_4)$$

$$m(v_1^2 - v_3^2) = M(V_4^2 - v_2^2)$$

$$\Rightarrow m(v_1 - v_3)(v_1 + v_3) = M(V_4 - V_2)(V_4 + V_2)$$

$$\Rightarrow (v_1 - v_3) = (V_4 - V_2)$$

(Now you can solve a linear equation)

$$\underbrace{v_1 + v_2}_{V_{rel} \text{ before collision}} = \underbrace{V_4 + V_3}_{V_{rel} \text{ after collission}}$$

(always holds for elastic collisions)

For less than elastic collissions, we can define a coefficient of restition as

$$V'_{rel} = e\, V_{rel}$$

COR

for completely inellastic collisions, e = 0

2) 

Find final velocities of the bodies for coefficient of restitution 'e'.

$$m_1 v_1 + m_2 v_2 = m_1 v_3 + m_2 v_4$$

$$e(v_1 - v_2) = v_4 - v_3$$

$$m_1 v_1 + m_2 v_2 + m_1 e(v_1 - v_2) = (m_1 + m_2) v_4$$

$$\Rightarrow v_4 = \frac{v_1(m_1 + e m_1) + v_2(m_2 - e m_1)}{m_1 + m_2}$$

$$m_1 v_1 + m_2 v_2 - m_2 e(v_1 - v_2) = (m_1 + m_2) v_3$$

$$\Rightarrow v_3 = \frac{v_1(m_1 - e m_2) + v_2(m_2 + e m_2)}{m_1 + m_2}$$

Check for e = 0 & e = 1

3) A tennis ball with a small mass 'm_2' sits on topa basketball with a large mass 'm_1'. The bottom of the basketball is a height 'h' above the ground and the bottom of the tennis ball is a height 'h+d' above the ground. The ball are dropped. To what height does the tennis ball bounce.

Assume $m_1 >> m_2$
instantaneous bounce and ellastic collission.
& small separation between balls.

→ Velocity of basketball $v = \sqrt{2gh}$

Right after bounce, it goes upward with $\sqrt{2gh}$

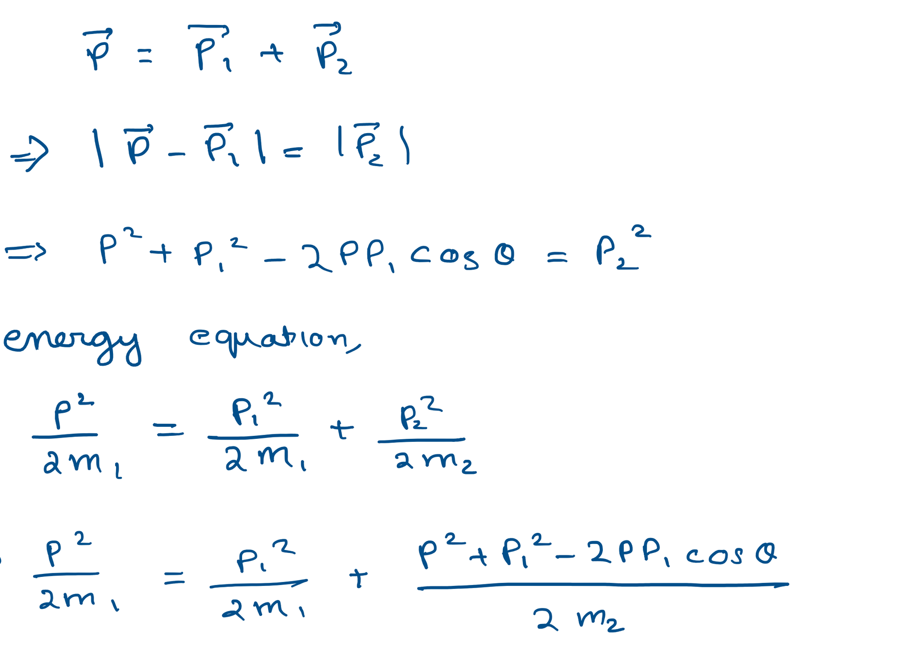
The tennis ball is moving downwards with $\sqrt{2gh}$

As relative velocity is 2v before collision, the tennis ball moves upwards with v+2v.

$$\therefore \text{ height reached} = d + \frac{(3v)^2}{2g}$$

$$= d + 9h$$

4) A particle of mass m_1 collides ellastically with a particle of mass m_2. Find the maximum angle through which the striking particle may deviate as a result of the collission.



By momentum conservation

$$\vec{P} = \vec{P_1} + \vec{P_2}$$

$$\Rightarrow |\vec{P} - \vec{P_1}| = |\vec{P_2}|$$

$$\Rightarrow P^2 + P_1^2 - 2PP_1\cos\theta = P_2^2$$

Using energy equation,

$$\frac{P^2}{2m_1} = \frac{P_1^2}{2m_1} + \frac{P_2^2}{2m_2}$$

$$\Rightarrow \frac{P^2}{2m_1} = \frac{P_1^2}{2m_1} + \frac{P^2 + P_1^2 - 2PP_1\cos\theta}{2m_2}$$

$$\Rightarrow P^2\left(\frac{1}{m_1} - \frac{1}{m_2}\right) = P_1^2\left(\frac{1}{m_1} + \frac{1}{m_2}\right) - \frac{2PP_1\cos\theta}{m_2}$$

$$\Rightarrow P^2\left(\frac{m_2 - m_1}{m_1 m_2}\right) + \frac{2PP_1\cos\theta}{m_2} - P_1^2\left(\frac{m_1 + m_2}{m_1 m_2}\right) = 0$$

For P to have a real solution,

$$(2P_1\cos\theta)^2 + 4\frac{(m_2 - m_1)}{m_1} P_1^2 \frac{(m_1 + m_2)}{m_1} \geq 0$$

$$\cos^2\theta + \frac{m_2^2 - m_1^2}{m_1^2} \geq 0$$

$$\left(\frac{m_2^2}{m_1^2}\right) \geq \sin^2\theta$$

$$\Rightarrow \sin\theta \leq m_2/m_1$$

5) A flatcar of mass m_0 starts moving to the right due to a constant horizontal force F. Sand spills on the flatcar from a stationary hopper. The velocity of loading is constant and equal to μ kg/s. Find the time dependence of the velocity and acceleration of the flatcar in the process of loading.

At time t

$$F_{net} = F - V\frac{dm}{dt}$$

$$m_{tot}\, a = F - v\mu$$

$$\Rightarrow (m_0 + \mu t)\, a = F - v\mu$$

$$\Rightarrow \int_0^V \frac{dv}{F - v\mu} = \int_0^t \frac{dt}{m_0 + \mu t}$$

$$\Rightarrow \ln\frac{F}{F - \mu V} = \ln\frac{m_0 + \mu t}{m_0}$$

$$\Rightarrow \frac{m_0 F}{m_0 + \mu t} = F - \mu V$$

$$\Rightarrow v = Ft/(m_0 + \mu t)$$

$$\& \quad a = \frac{F}{(m_0 + \mu t)} - \frac{Ft(\mu)}{(m_0 + \mu t)^2}$$

$$= \frac{F m_0}{(m_0 + \mu t)^2}$$