Lecture 5

Thursday, 17 October 2019 9:34 AM

Newton's role in the history of humankind is unparalled. Not only in science but in the general nature of discourse. It was afterall the 'universal' nature of gravitation that made the laws of earth and heavens the same.

Astronomy told us that

$$F_G \propto \frac{1}{r^2}$$

$$\propto M_G$$

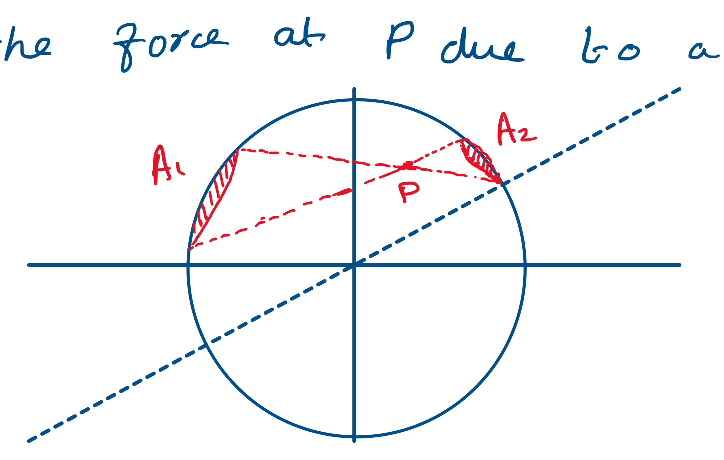
$$\propto m_G$$

$$\therefore F_G = -\frac{G M_G m_G}{r^2} \hat{r}$$

The Eötvös experiment confirmed that $m_G = m_I$ (inertial) upto a current factor of 10^{-13}

Now that we know what the force is supposed to look like, lets solve a very essential result.

1. Find the force at P due to a mass shell.

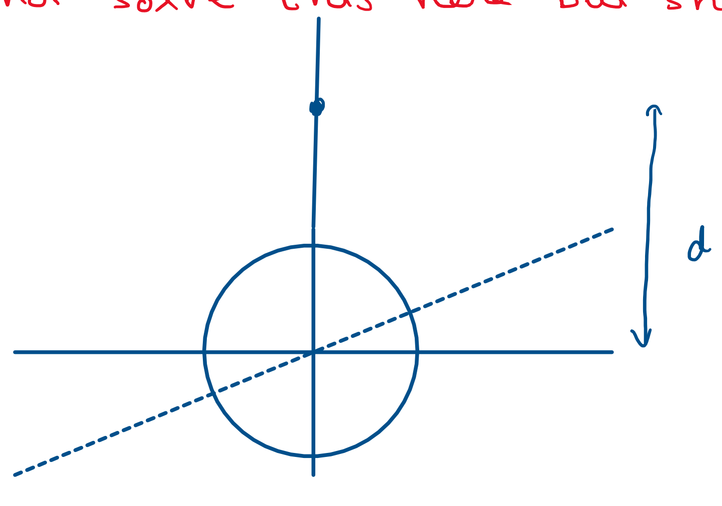


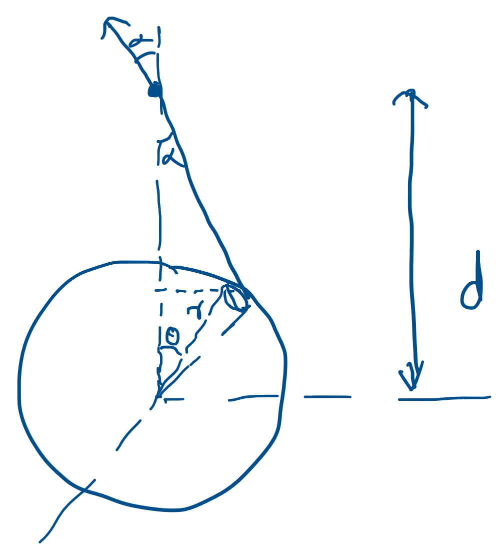
$$F = \frac{Gm(\rho A_1 t)}{r_1^2} - \frac{G(\rho A_2 t)m}{r_2^2}$$

$$= G\rho t m \left(\frac{A_1}{r_1^2} - \frac{A_2}{r_2^2}\right) \qquad A_1 \propto r_1^2 \quad A_2 \propto r_2^2$$

$$= 0$$

2. Find the gravitational force of a mass shell at a distance d.
(We will not solve this here but show you how it is done)





$$dF = -\frac{Gm\,\sigma(R\,d\theta\,R\sin\theta\,d\phi)}{(R^2 + d^2 - 2Rd\cos\theta)} \cos(\alpha)$$

$$= -\frac{Gm\sigma(R^2\sin\theta\,d\theta\,d\phi)}{(R^2 + d^2 - 2Rd\cos\theta)^{3/2}} (d - R\cos\theta)$$

$$F = -\frac{GMm}{d^2}$$

(This is more easily solved using potential. We will wait to solve this till we reach it)

3) Find the gravitational force on a unit mass due a volume of mass density at a distance 'r' from the center.

$r < R$

$$F_{(r)} = \frac{\int_R^r dM}{r^2} G$$

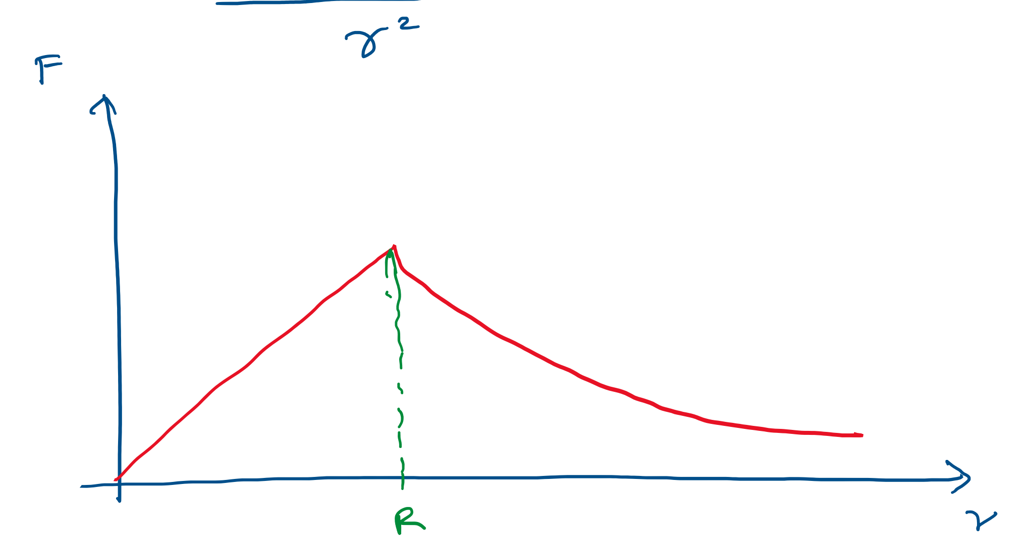
$$= \frac{G \rho \frac{4}{3}\pi r^3}{r^2}$$

$$= \frac{4 G\rho\pi r}{3}$$

$r > R$

$$F(r) = \frac{\rho \times 4/3 \pi R^3 G}{r^2}$$

$\therefore$



4) Find the force due to gravitation at a depth d & height h from the Earth's surface. when $d, h \ll R$

$$F = \frac{GMm}{R^2} \quad \text{(at the surface)}$$

$$F = \frac{GMm}{(R+h)^2} \quad \text{(at height h above surface)}$$

$$= \frac{GMm}{R^2}(1 + h/R)^{-2}$$

$$= \frac{GMm}{R^2}(1 - 2h/R)$$

$$F = \frac{G\left(\frac{4}{3}\pi (R-d)^3 \rho\right) m}{(R-d)^2} \quad \text{(at distance d below surface)}$$

$$= \frac{G\left(\frac{4}{3}\pi R^3 \rho\right) m}{R^2}(1 - d/R)$$

$$= \frac{GMm}{R^2}(1 - d/R)$$

5) A straight tunnel is drawn from N.Y. to S.F. which has a distance of 5000 km as measured from the surface. A car rolling on steel rails is released from rest at N.Y. and reaches S.F. through the tunnel.

i) Neglecting friction and earth's rotation, how long does it take to get there.

ii) Suppose there is friction now, how does it affect the motion? Let the friction be proportional to square of velocity.

iii) Now consider the effects of rotation. Estimate the magnitude of centrifugal and coriolis force relative to the gravitational force.