Lecture 4

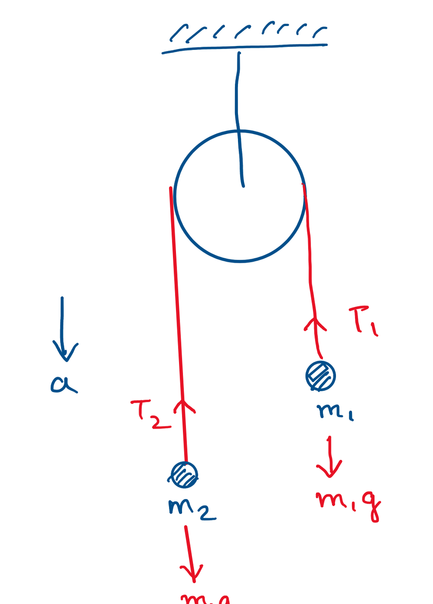
Monday, 14 October 2019 8:41 AM

We have more problems to solve today. In the next class we shall move onto more complex geometries.

1) A fixed pulley carries a massless thread with masses m_1 & m_2 at its ends. There is friction between the thread and pulley. It starts slipping when the ratio $m_2/m_1 = \eta_0$

a) Find the friction coefficient.
b) The acceleration of the masses when $m_2/m_1 > \eta_0$

a)

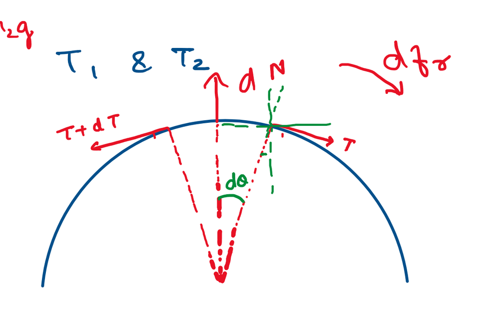


$$T_1 - m_1 g = m_1 a$$

$$m_2 g - T_2 = m_2 a$$

$$\Rightarrow \frac{T_2}{T_1} = \frac{m_2 (g-a)}{m_1 (g+a)}$$

We need to relate T_1 & T_2



$$(T+dT)\sin(d\theta/2) + T\sin(d\theta/2) = dN$$

$$\Rightarrow T\,d\theta = dN$$

$$\therefore df_r = \mu\, dN = \mu T\, d\theta \quad \text{....(i)}$$

Also

$$(T+dT)\cos(d\theta/2) - T\cos(d\theta/2) - df_r = 0$$

$$\Rightarrow dT = df_r$$

$$\Rightarrow dT = \mu T\, d\theta$$

$$\int_{T_1}^{T_2} \frac{dT}{T} = \int_0^{\pi} \mu\, d\theta \Rightarrow \ln(T_2/T_1) = \mu\pi$$

$$\Rightarrow T_2/T_1 = e^{\mu\pi}$$

$$e^{\mu\pi} = \frac{m_2}{m_1}\frac{(g-a)}{(g+a)} = \eta\left[\frac{g-a}{g+a}\right] \quad \text{....(ii)}$$

At $\eta = \eta_0$, $a = 0$

$$e^{\mu\pi} = \eta_0$$

$$\Rightarrow \mu = \frac{1}{\pi}\ln\eta_0$$

b) $$e^{\ln\eta_0} = \eta\left[\frac{g-a}{g+a}\right]$$

$$\Rightarrow \frac{\eta_0}{\eta} = \frac{g-a}{g+a}$$

$$\Rightarrow a = g\frac{[\eta - \eta_0]}{[\eta + \eta_0]}$$

2) A motorboat of mass 'm' moves along a lake with velocity v_0. At the moment $t=0$, the engine of the boat is shut down. Assuming the resistance of water to be proportional to the velocity of the boat $\vec{F} = -\gamma\vec{v}$, find

a) How long the motorboat moved with the shutdown engine?
b) The velocity of the motorboat as a function of the distance travelled
c) What is the total distance covered?
d) The mean velocity of the boat over the time interval its initial velocity decreases by η times.

a) $$F = -\gamma v$$

$$\Rightarrow \frac{dv}{dt} = -\frac{\gamma}{m}v$$

$$\Rightarrow \int_{v_0}^{v}\frac{dv}{v} = -\frac{\gamma}{m}\int_0^t dt$$

$$\Rightarrow \ln(v/v_0) = -\gamma/m\, t$$

$$\Rightarrow v = v_0 e^{-\gamma/m\, t}$$

v will be zero at $t \to \infty$

b) $$a = -\gamma/m\, v$$

$\left[\text{Now } a = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds} \right]$

$$\Rightarrow \cancel{v}\frac{dv}{ds} = -\frac{\gamma}{m}\cancel{v}$$

$$\Rightarrow \int_{v_0}^{v} dv = -\frac{\gamma}{m}\int_0^s ds$$

$$\Rightarrow v - v_0 = -\frac{\gamma}{m}s$$

$$\Rightarrow v = v_0 - \frac{\gamma}{m}s$$

c) Total distance travelled, $v = 0$

$$\Rightarrow s = \frac{m v_0}{\gamma}$$

d) $$\frac{v_0}{\eta} = v_0 - \frac{\gamma}{m}s' \Rightarrow s' = \frac{m v_0}{\gamma}\left(\frac{\eta-1}{\eta}\right)$$

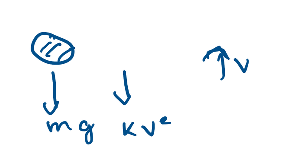
& $$\frac{v_0}{\eta} = v_0 e^{-\gamma/m\, t'} \Rightarrow t' = \frac{m}{\gamma}\ln(\eta)$$

$$\langle v \rangle = \frac{s'}{t'} = \frac{v_0(\eta-1)}{\eta\ln\eta}$$

3) A body of mass 'm' is thrown straight up with velocity v_0. Find the velocity with which the body comes down if the air drag equals 'kv^2', where 'k' is constant & 'v' is the velocity of the body.

<u>Going Upward</u>

$$F_{net} = mg + kv^2$$



$$\Rightarrow a = g + \frac{kv^2}{m}$$

$$\Rightarrow -v\frac{dv}{ds} = \frac{mg + kv^2}{m}$$

$$\Rightarrow \int_{v_0}^{0}\frac{mv\,dv}{(mg+kv^2)} = -\int_0^h ds$$

$$\Rightarrow h = \frac{m}{2k}\ln\left(\frac{kv_0^2 + mg}{mg}\right)$$

<u>Going Downward</u>

$$F_{net} = mg - kv^2$$

$$\Rightarrow a = g - \frac{kv^2}{m}$$

$$\Rightarrow v\frac{dv}{ds} = \left(g - \frac{kv^2}{m}\right)$$

$$\Rightarrow \int_0^v \left(\frac{mv\,dv}{mg - kv^2}\right) = \int_0^h ds$$

$$h = \frac{m}{2k}\ln\left(\frac{mg}{mg - kv^2}\right)$$

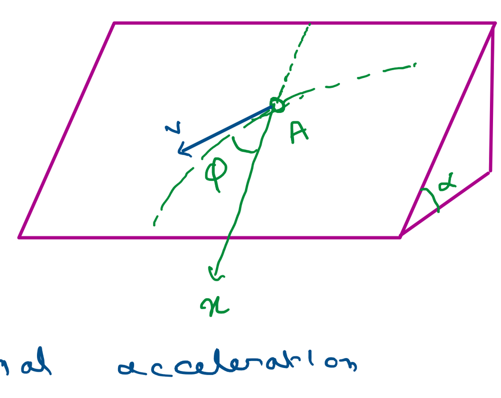
$$\therefore \frac{m}{2k}\ln\left(\frac{mg}{mg - kv^2}\right) = \frac{m}{2k}\ln\left(\frac{mg + kv_0^2}{mg}\right)$$

$$\Rightarrow \cancel{m^2g^2} = \cancel{m^2g^2} + kv_0^2 mg - kv^2(mg + kv_0^2)$$

$$\Rightarrow v^2 = \frac{v_0^2 mg}{(mg + kv_0^2)}$$

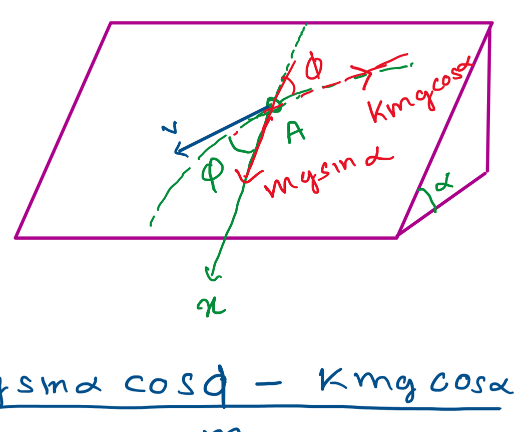
$$\Rightarrow v = v_0 \Big/ \sqrt{1 + kv_0^2/mg}$$

4) A small disc 'A' is placed on an inclined plane forming an angle 'α' with the horizontal and is imparted an initial velocity v_0. Find how the velocity of the disc depends on the angle φ, if the friction is $k = \tan\alpha$ and the initial moment $\varphi_0 = \pi/2$



a_z = tangential acceleration

a_x = acceleration in the x-direction.



$$a_z = \frac{mg\sin\alpha\cos\phi - kmg\cos\alpha}{m}$$

$$= g\,(\sin\alpha\cos\phi - \sin\alpha]$$

$$= g\sin\alpha\,[\cos\phi - 1]$$

$$a_x = mg\sin\alpha - k\,mg\cos\alpha\cos\phi$$

$$= g\sin\alpha\,[1 - \cos\phi]$$

$$\therefore a_z = -a_x$$

$$\Rightarrow \int\frac{dv_z}{dt} = -\int\frac{dv_x}{dt}$$

$$\Rightarrow v_z = -v_x + C \quad \text{at } t=0,\ v_z = v_0 \ \&\ v_x = 0$$

$$\therefore C = v_0$$

$$\therefore v_z = -v_x + v_0$$

& $$v_x = v_z\cos\phi$$

$$\therefore v_z = v_0/(1+\cos\phi)$$