Lecture 3

Thursday, 10 October 2019 7:57 AM

Newton's Laws

i) objects retain their state of motion
→ (imagine a body at rest in one frame and then move into another)

ii) rate of change in momentum is proportional to the force applied

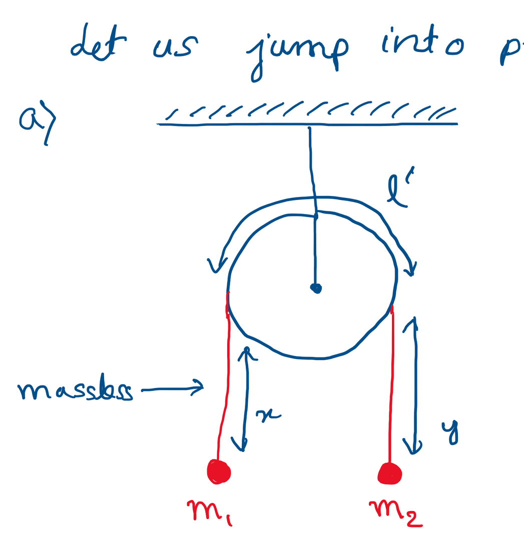
$$\vec{F} = m\vec{a}$$

(m ← inertial mass of body; $\vec{a}$ ← acceleration of object; $\vec{F}$ ← sum of all forces)

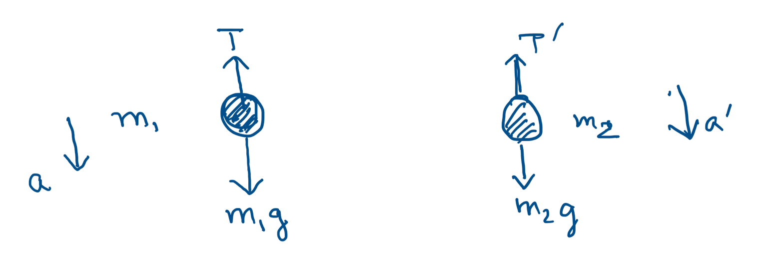
iii) $\vec{F}_{12} = -\vec{F}_{21}$

Let us jump into problems immediately.

a)



FBDs & Constraints



$$m_1 g - T = m_1 a$$

$$m_2 g - T' = m_2 a'$$

Now, $T = T'$ or else the string has infinite acceleration

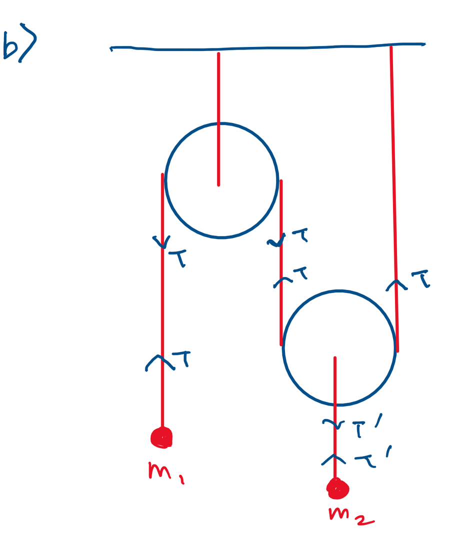
& $x + l' + y = l$ ← total length of string

$$\Rightarrow \ddot{x} + \ddot{y} = 0$$

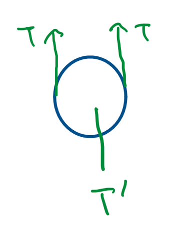
$$\Rightarrow \vec{a} = -\vec{a}'$$

$$\therefore a = \frac{m_1 g - m_2 g}{m_1 + m_2}, \quad >0 \text{ if } m_1 > m_2, \quad <0 \text{ if } m_1 < m_2$$

b)

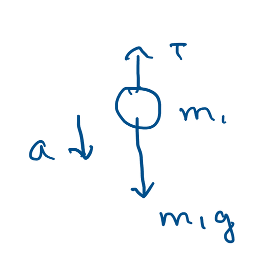


Find acceleration of m_1 & m_2

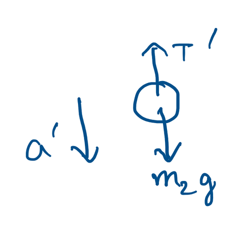


As pulley is massless

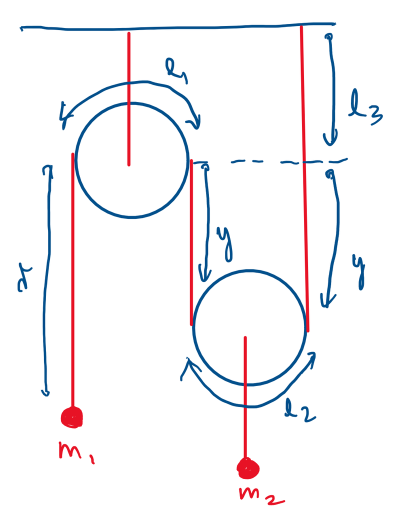
$$2T - T' = 0$$



$$m_1 g - T = m_1 a$$



$$m_2 g - T' = m_2 a'$$



$$x + l_1 + y + l_2 + y + l_3 = l$$

$$\Rightarrow \dot{x} = -2\dot{y}$$

$$\Rightarrow a = -2a'$$

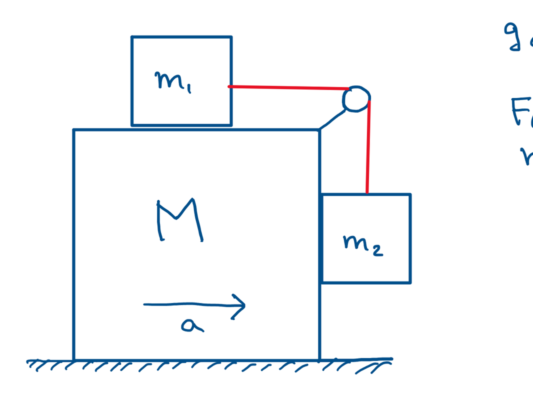
$$\therefore m_1 g - T = m_1 a$$

$$\Rightarrow m_2 g - 2T = -\frac{m_2 a}{2}$$

$$\Rightarrow a = \frac{4m_1 g - 2m_2 g}{4m_1 + m_2}$$

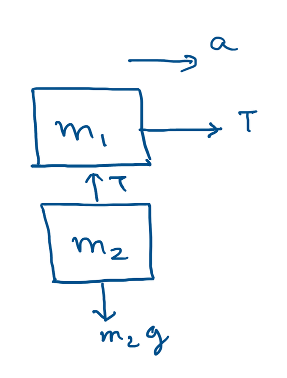
$$\& \quad a' = \frac{2m_1 g - m_2 g}{4m_1 + m_2}$$

c) Now that we know how pulleys work we can turn to more exciting ones



Ignore all friction

For what value 'a' will the masses m_1 & m_2 not move relative to M?



$$T = m_1 a$$

$$T = m_2 g$$

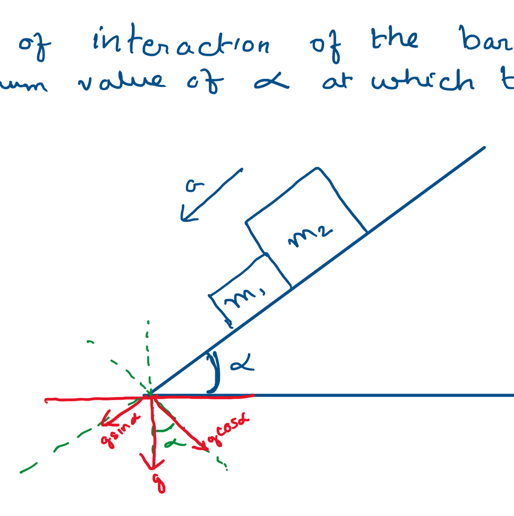
$$\therefore a = \frac{m_2}{m_1} g$$

d) Two bars 1 & 2 are placed on an inclined plane forming an angle 'α' with the horizontal. The masses of bars are equal to m_1 & m_2 and the coefficients of friction are k_1 & k_2. ($k_1 > k_2$).

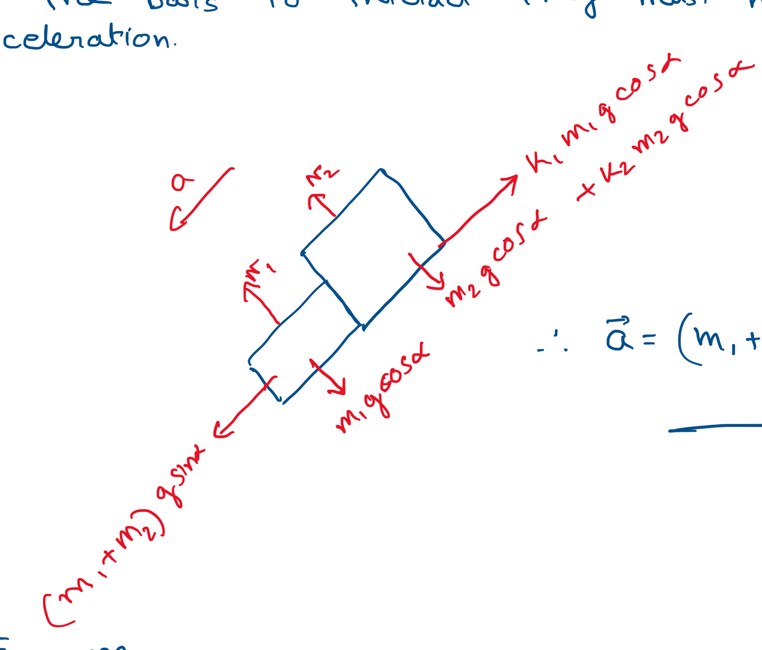
Find

i) force of interaction of the bars in the process of motion.

ii) minimum value of α at which the bars start sliding down

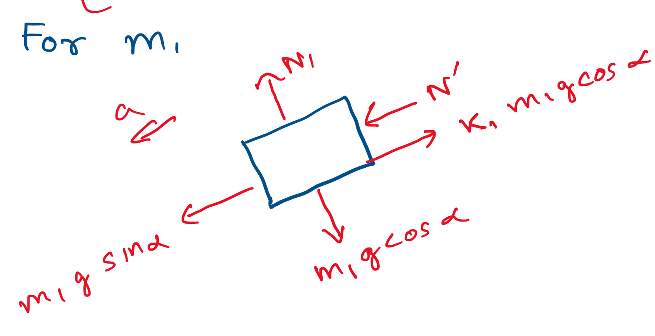


For the bars to interact they must move with the same acceleration.



$$\therefore \vec{a} = \frac{(m_1+m_2)g\sin\alpha - (k_1 m_1 + k_2 m_2)g\cos\alpha}{(m_1+m_2)}$$

For m_1



$$m_1 g\sin\alpha + N' - k_1 m_1 g\cos\alpha = m_1 a$$

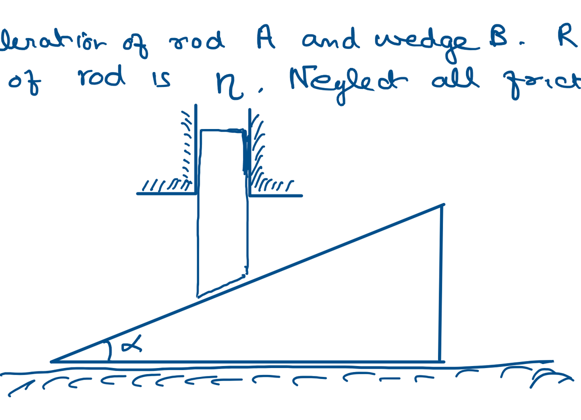
$$\Rightarrow N' = k_1 m_1 g\cos\alpha - m_1 g\sin\alpha + \frac{m_1\left((m_1+m_2)g\sin\alpha - (k_1 m_1 + k_2 m_2)g\cos\alpha\right)}{(m_1+m_2)}$$

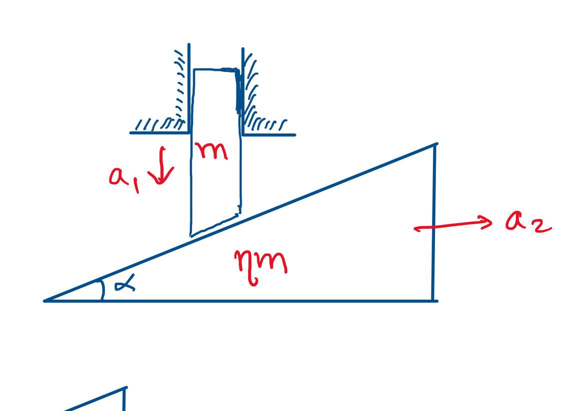
$$= \frac{(k_1 - k_2)m_1 m_2 \cos\alpha}{(m_1+m_2)}$$

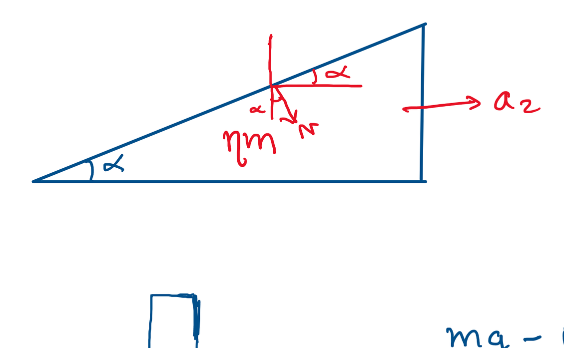
ii) For minimum value of 'α' acceleration should be zero.

$$\therefore \tan\alpha = \frac{k_1 m_1 + k_2 m_2}{k_1 + k_2}$$

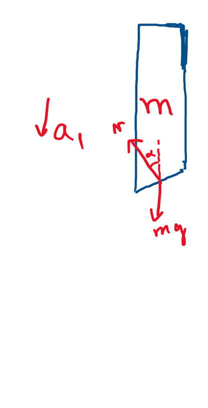
e) Find the acceleration of rod A and wedge B. Ratio of mass of wedge to that of rod is η. Neglect all friction.



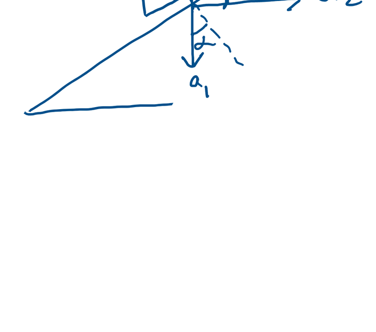




$$N\sin\alpha = \eta m a_2$$



$$mg - N\cos\alpha = m a_1$$

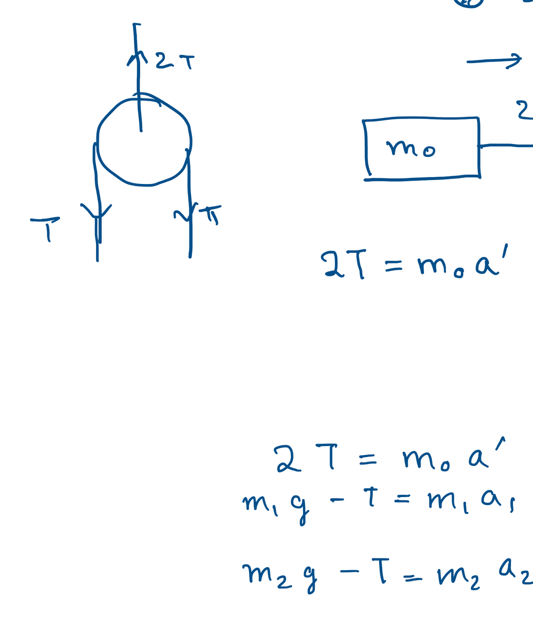


$$a_2 \sin\alpha = a_1 \cos\alpha$$

$$a_1 = \frac{g}{1 + \eta\cot^2\alpha}$$

$$a_2 = \frac{g}{\tan\alpha + \eta\cot\alpha}$$

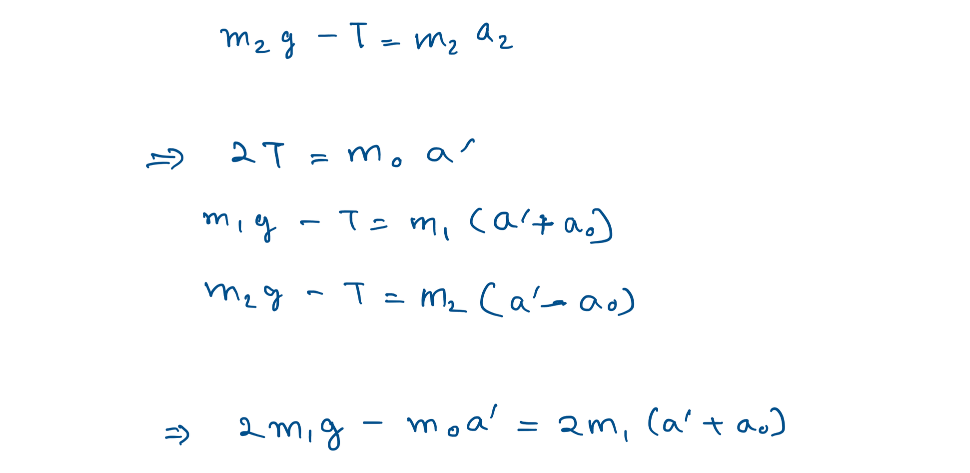
f)



Find the acceleration of m_0, m_1, m_2.

Massless & unstretchable string.

Frictionless surfaces.



$$2T = m_0 a'$$

$$m_1 g - T = m_1 a_1$$

$$m_2 g - T = m_2 a_2$$

$$2T = m_0 a'$$
$$m_1 g - T = m_1 a_1$$
$$m_2 g - T = m_2 a_2$$

$$\Rightarrow 2T = m_0 a'$$
$$m_1 g - T = m_1 (a' + a_0)$$
$$m_2 g - T = m_2 (a' - a_0)$$

$$\Rightarrow 2m_1 g - m_0 a' = 2m_1 (a' + a_0)$$
$$2m_2 g - m_0 a' = 2m_2 (a' - a_0)$$

$$\Rightarrow a' = \frac{4m_1 m_2 g}{m_0(m_1+m_2) + 4m_1 m_2}$$

$$(a' + a_0) = g - \frac{m_0}{2m_1} a'$$

$$= g - \frac{2m_2 m_0 g}{m_0(m_1+m_2) + 4m_1 m_2}$$

$$= \frac{4m_1 + m_2 + m_0(m_1 - m_2)g}{m_0(m_1+m_2) + 4m_1 m_2}$$