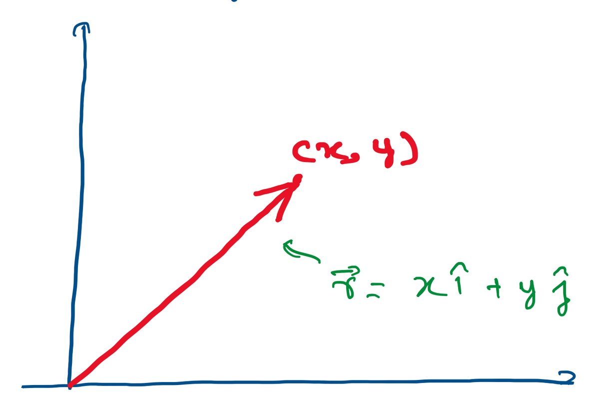
Lecture 2

Monday, 7 October 2019 9:10 AM

In the last class, we discussed briefly about vectors and their algebra. In this class, we will rely on our lessons from then & talk in the language of vectors. Calculus will be introduced as we encounter it.
[The new comers can breathe easy. We will repeat almost every topic from the previous class]

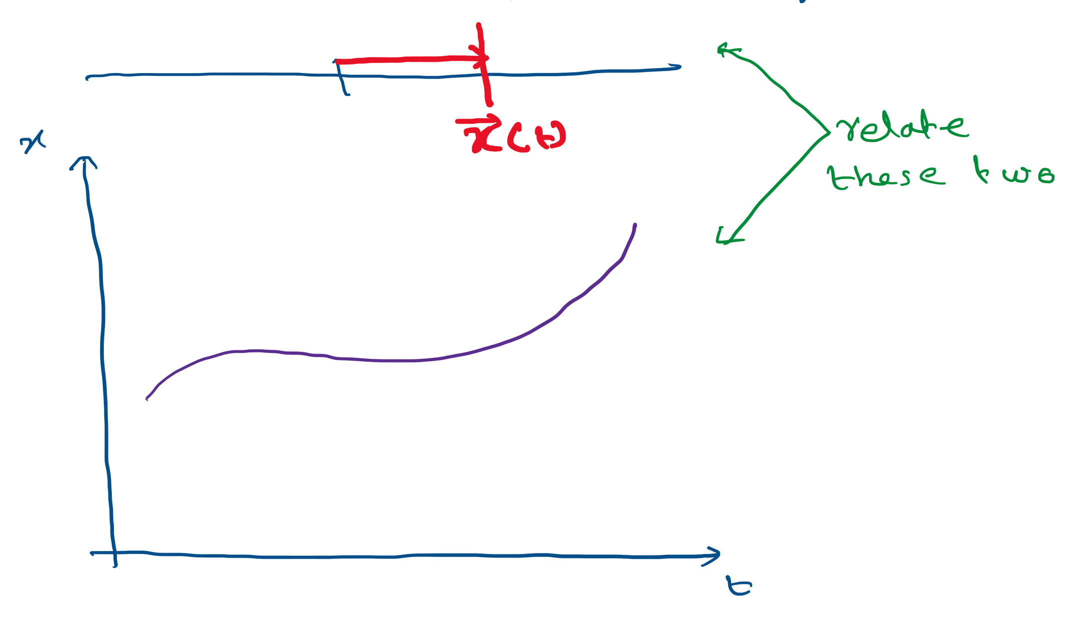
As we have discussed earlier, a position in space can be described using a vector.

Let's say



But of course this static vector is not interesting. Sure we know where an object is in space now. But objects move. How do we denote that.

Let us take the more simplistic case of 1D vectors



The immediate question arises about how do we study this motion. The intuitive argument suggest that we want to know how fast is the body moving and if that speed is changing or not.

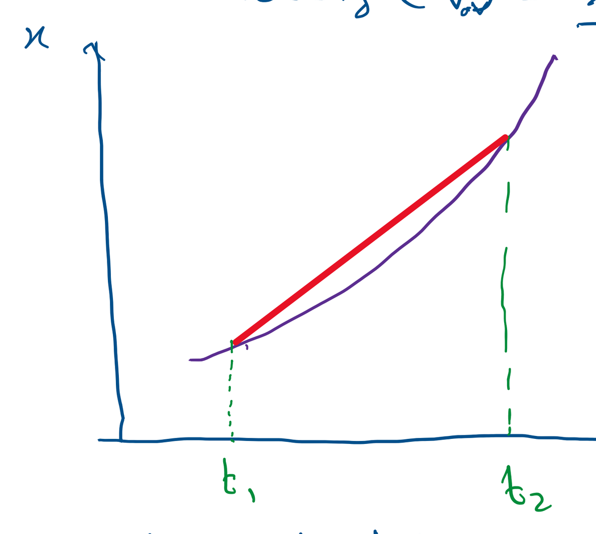
Using high school physics we know that,

$$\text{Speed} = \frac{x(t_2) - x(t_1)}{t_2 - t_1}$$

$$\text{velocity } (\vec{V}_{av}) = \frac{\vec{x}(t_2) - \vec{x}(t_1)}{t_2 - t_1}$$

makes more sense in higher dimensions

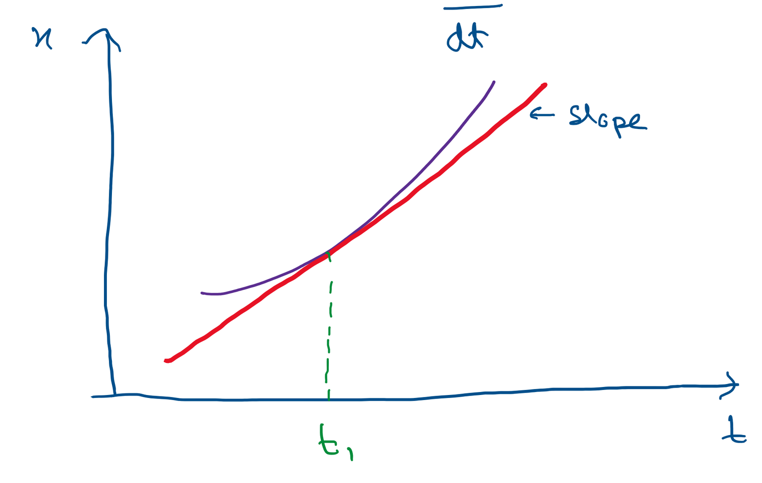
slope of line joining the two points



But we want to do better and describe what happens instantaneously.

$$\therefore \vec{V}_{in} = \vec{V} = \lim_{h \to 0} \frac{\vec{x}(t+h) - \vec{x}(t)}{(t+h) - t} = \frac{d\vec{x}}{dt}$$

$$\therefore \vec{V} = \frac{d\vec{x}}{dt} = \dot{x}$$



Using the same argument, if we had $v(t)$ we can find the instantaneous change in velocity as

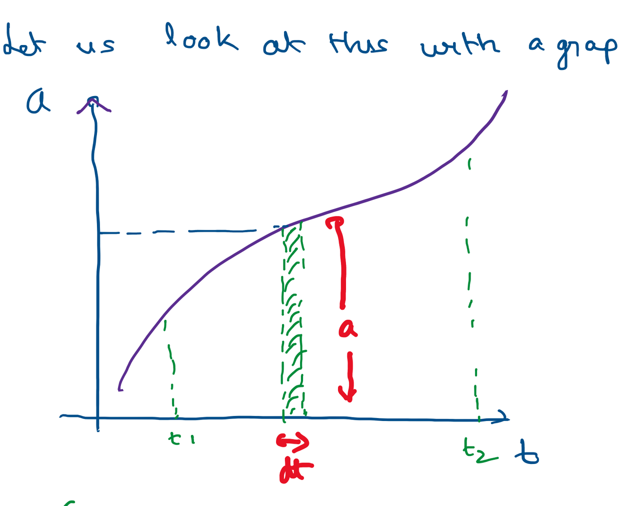
$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v} = \ddot{x}$$

This can go the other way too. Let's say you had a & wanted to find the change in velocity. How would you do that?

$$\text{As } \frac{d\vec{v}}{dt} = \vec{a}$$

$$\Rightarrow d\vec{v} = \vec{a}\, dt$$

Let us look at this with a graph,



$$dv = a(t)\, dt$$

$$\int_c^v dv = \int_{t_1}^{t_2} a(t)\, dt$$

$$\therefore v = \int_{t_1}^{t_2} a(t)\, dt + c$$

(Change in 'v' is area under the graph)

Essential 1D Kinematics equation (for constant 'a')

$$v = a\,t\Big|_0^t + c \Rightarrow \boxed{v = v_0 + at}$$

$$\frac{dx}{dt} = v \Rightarrow dx = v\,dt$$

$$\Rightarrow \int_{x_0}^{x} dx = \int_0^t v\,dt$$

$$\Rightarrow x = x_0 + \int_0^t (v_0 + at)\, dt$$

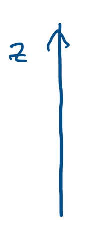
$$\Rightarrow \boxed{x = x_0 + v_0 t + \frac{1}{2} a t^2}$$

$$\boxed{v^2 = v_0^2 + 2a(x - x_0)}$$

Let us use these to look at a few problems.

1) You throw a ball upwards into the air with speed v_0. When does it return to your hand again. Take g to be constant.

[Always start by defining your coordinate system]



$$\therefore \vec{a} = -g\hat{k}$$

$$\vec{v}(0) = v_0\hat{k}$$

$$\vec{x}(0) = 0\hat{k}$$

$$x = 0 + v_0 t + \frac{1}{2}(-g)t^2$$

Take $x = 0$

$$\Rightarrow 0 = t\left(v_0 - \frac{gt}{2}\right)$$

$$\Rightarrow t = 0 \quad \& \quad t = \frac{2v_0}{g}$$

when it was thrown — when it returned

In higher dimensions, the problems remains essentially the same. One component doesn't affect the other.

Let us say $\vec{v}(0)$ was along $\hat{x}$ & $\hat{y}$ but 'a' was along $\hat{x}$ only

$$\therefore \vec{V}(0) = v_0\hat{i} + v_0\hat{j} \quad \& \quad \vec{a} = a_0\hat{i}$$

$$\therefore \vec{v}(t) = \int_0^t \vec{a}\, dt + \vec{v}(0)$$

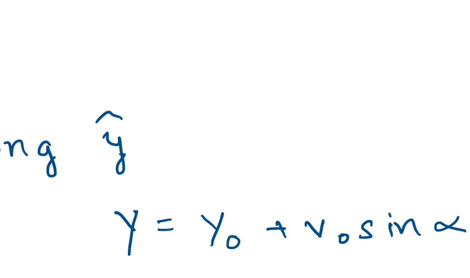
$$\Rightarrow \vec{v}(t) = a_0 t\,\hat{i} + v_0\hat{i} + v_0\hat{j}$$

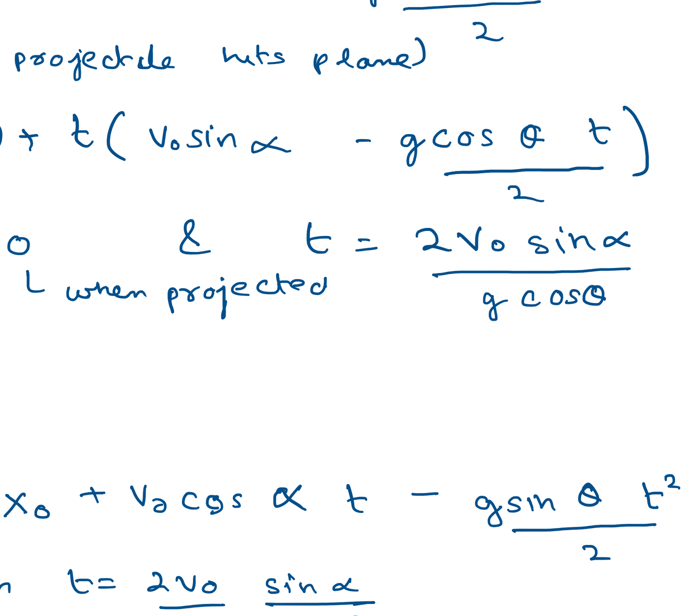
$$\Rightarrow \vec{v}(t) \text{ along } \hat{i} = a_0 t + v_0$$

$$\vec{v}(t) \text{ along } \hat{j} = v_0$$

Let us use this to solve,

2) a) When & how high does the projectile hit the inclined plane.





Along $\hat{y}$

$$y = y_0 + v_0 \sin\alpha\, t - \frac{g\cos\theta}{2} t^2$$

For $y = 0$ (projectile hits plane)

$$\Rightarrow 0 = 0 + t\left(v_0 \sin\alpha - \frac{g\cos\theta}{2} t\right)$$

$$\therefore t = 0 \quad \& \quad t = \frac{2v_0 \sin\alpha}{g\cos\theta}$$

└ when projected

Along x

$$x = x_0 + v_0 \cos\alpha\, t - \frac{g\sin\theta}{2} t^2$$

Plug in $t = \frac{2v_0}{g}\frac{\sin\alpha}{\cos\theta}$

$$\therefore x = v_0\cos\alpha \times \frac{2v_0}{g}\frac{\sin\alpha}{\cos\theta} - \frac{g\sin\theta}{2}\frac{4v_0^2}{g^2}\frac{\sin^2\alpha}{\cos^2\theta}$$

$$= \frac{2v_0^2}{g}\frac{\sin\alpha}{\cos\theta}\left(\cos\alpha - \sin\alpha\tan\theta\right)$$

[Check if this holds for $\theta = 0°$]

(*) Speed problem if there is time

Reference frames

"Laws of physics hold in all inertial frames"

└ For the complete significance of this line, see me again in Ph1b.

[There are no special frames.]

If an observer sees a projectile moving with velocity $\vec{v}$ from their frame, from the ground the frame

$$\boxed{\vec{V}_{pg} = \vec{V}_{po} + \vec{V}_{og}}$$

$\vec{V}_{og}$ → velocity of observer in ground frame

$\vec{V}_{po}$ → velocity of projectile in observer frame

$\vec{V}_{pg}$ → velocity of projectile in ground frame

(*) A travels at '40 m/s' from North to South & B travels from east to west with '30 m/s'. Find V_{AB} ← velocity of A as seen by B.

$$\vec{V}_{AG} = \vec{V}_{AB} + \vec{V}_{BG}$$

$$\boxed{\vec{V}_{AB} = \vec{V}_{AG} - \vec{V}_{BG}}$$

$$= (-40\hat{j}) - (-30\hat{i})$$

$$= 30\hat{i} - 40\hat{j}$$