Lecture 16

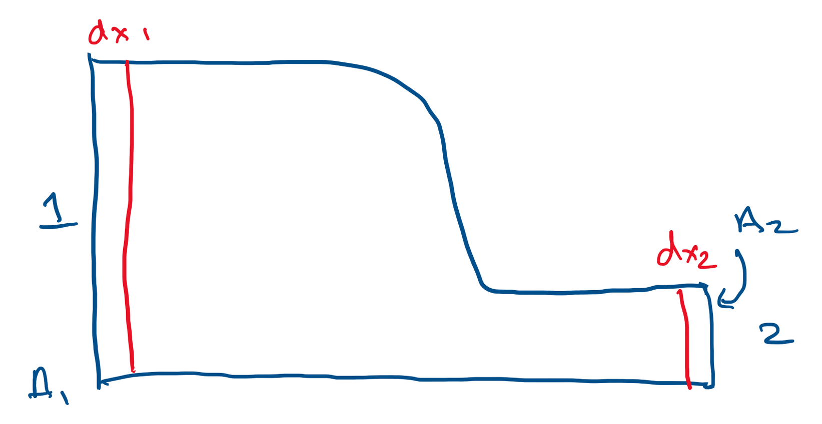
Sunday, 24 November 2019 4:16 PM

Real fluids are really difficult to work with.

For this course, we will make our life easier and consider
i) fluids are non-viscous - doesnt loose energy in motion
ii) flow is steady - the velocity at a point in space doesnt vary.

iii) fluid is incompressible - density is invariable.
iv) fluid does not rotate - No eddies or vortexes

Let us see how we can use these considerations to analyse a fluid.



If the velocity is v_1 at 1, what is the velocity at 2?

Let us assume that over some time dt, a particular amount of fluid enters the pipe. The same must leave. (incompressible)

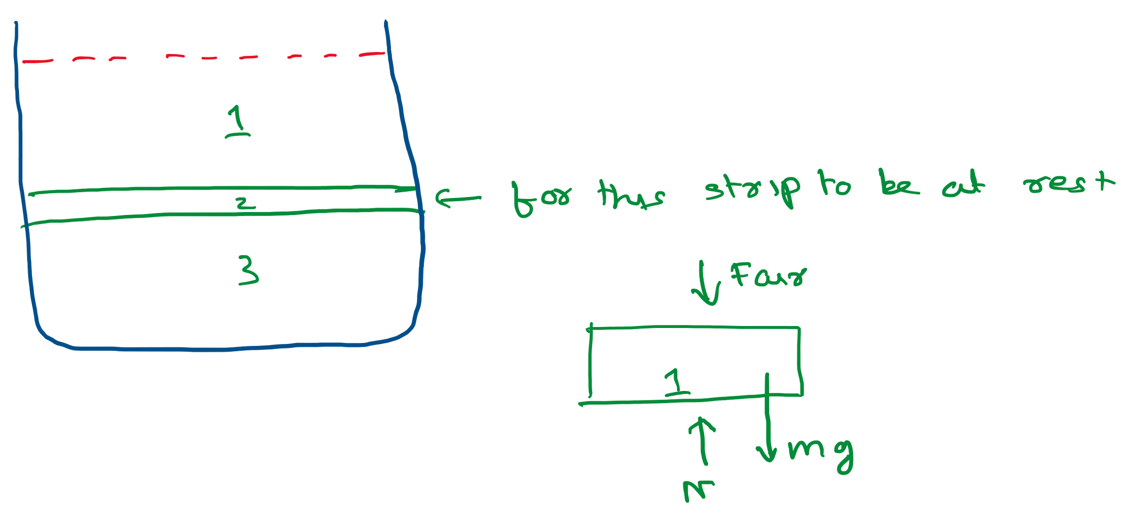
$\therefore$

$$\rho A_1 dx_1 = \rho A_2 dx_2$$

$$\Rightarrow A_1 \frac{dx_1}{dt} = A_2 \frac{dx_2}{dt}$$

$$\boxed{\Rightarrow A_1 v_1 = A_2 v_2}$$

Now let us consider a fluid at rest in a jar



(Argue why all points on the plane have the same pressure)

(Discuss rotating bucket)

$$\therefore N = mg + F_{air}$$

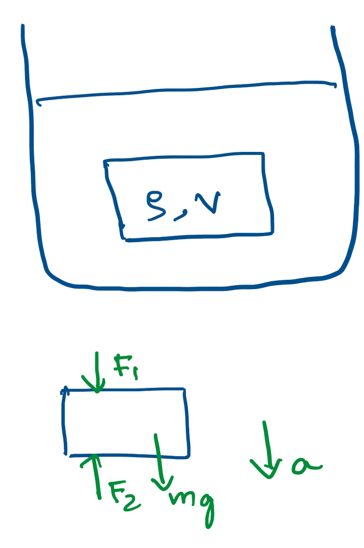
$$\Rightarrow P_2 = \frac{mg}{A} + P_{air}$$

$$\Rightarrow P_2 = \frac{\rho A h g}{A} + P_{air}$$

$$\Rightarrow P_2 = \rho g h + P_{air}$$

(pressure increases as you go deeper in the fluid)

Lets use what we learned for a body dipped in our fluid.



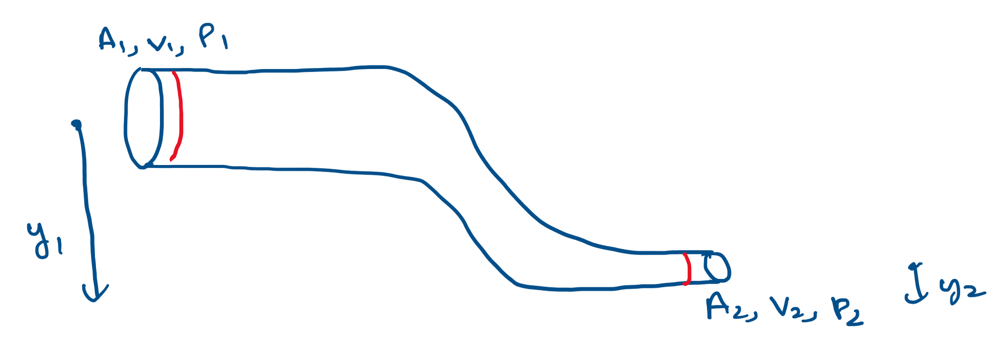
$$ma = mg + F_1 - F_2$$

$$\Rightarrow ma = mg + (P_1 - P_2)A$$

$$\Rightarrow ma = mg + \underbrace{\rho_w g h A}_{\text{Weight of water displaced}}$$

(Force of Buoyancy)

How about taking this a step further and figuring out how energy is associated to fluids?



$$\text{Work done} = P_1 \underbrace{A_1 dx_1}_{V} - P_2 \underbrace{A_2 dx_2}_{V} = P_1 V - P_2 V$$

$$\text{Kinetic Energy Lost} = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$\text{Potential Energy Lost} = mgy_2 - mgy_1$$

Energy Gained = Energy Lost

$$P_1 A_1 dx_1 - P_2 A_2 dx_2 = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 + mgy_2 - mgy_1$$

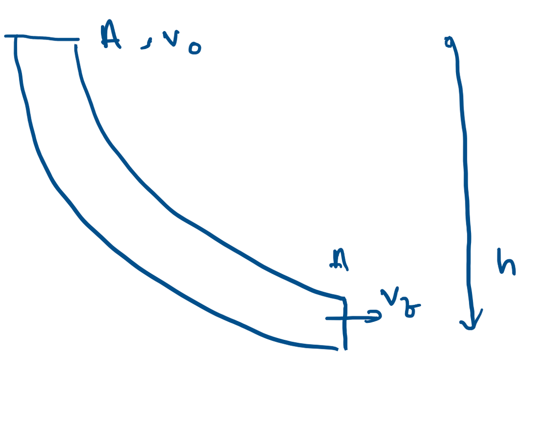
$$\Rightarrow P_1 V - P_2 V = \frac{1}{2} \rho V v_2^2 - \frac{1}{2} \rho V v_1^2 + \rho V g y_2 - \rho V g y_1$$

$$\Rightarrow P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2$$

$$\therefore P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = \text{constant}.$$

1) Fluid flows through a hose of uniform area A. It comes in at velocity v_0. What is the final velocity? Explain how it makes sense.

As the fluid density doesnt change



$$A v_0 = A v_f$$

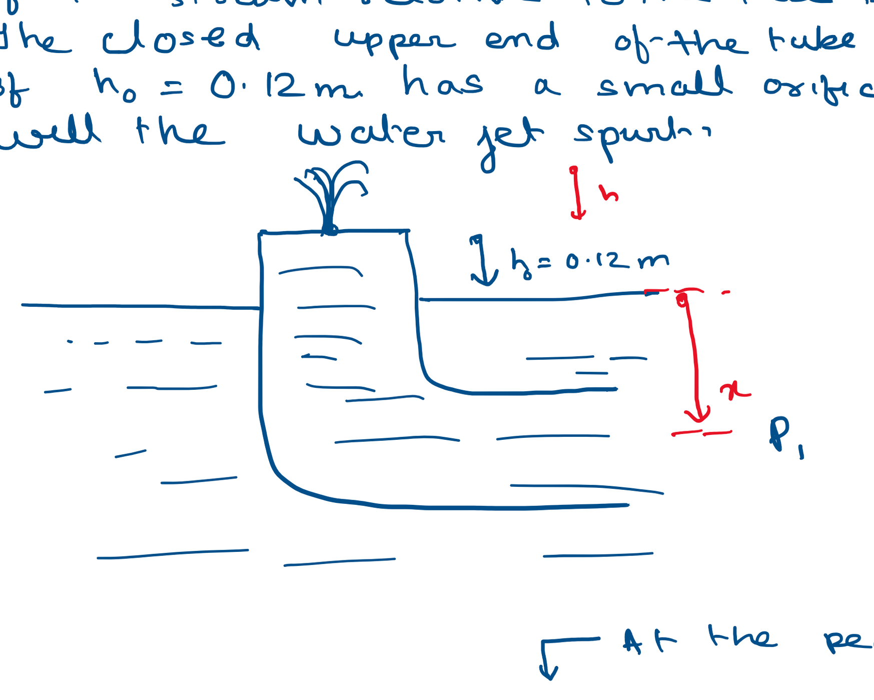
$$\therefore v_f = v_0$$

But what about the energy lost in gravitational potential.

$$P + \rho g h = \text{constant}$$

$\therefore$ pressure increases.

2) A bent tube is lowered into a water stream. The velocity of the stream relative to the tube is equal to $v = 2.5$ m/s. The closed upper end of the tube located at a height of $h_0 = 0.12$ m has a small orifice. To what height will the water jet spurt?



At the peak of the jet

$$P_1 + \frac{1}{2} \rho v_1^2 = P_0 + 0 + \rho g (h + h_0 + x)$$

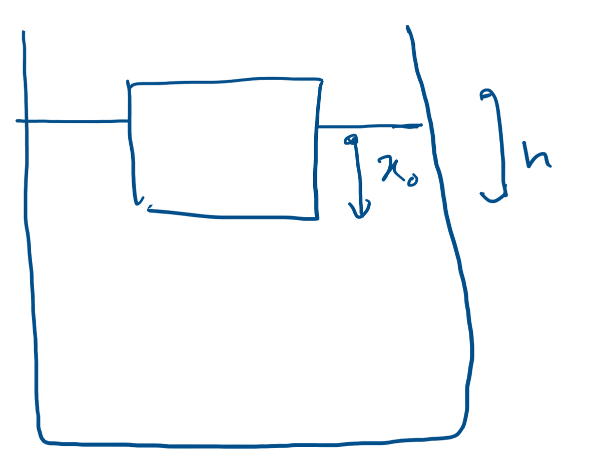
$$\& \quad P_1 = P_0 + \rho g x$$

$$\Rightarrow \cancel{P_0} + \cancel{\rho g x} + \frac{1}{2} \cancel{\rho} v_1^2 = \cancel{P_0} + \cancel{\rho} g (h + h_0) + \cancel{\rho g x}$$

$$\Rightarrow (h + h_0) = \frac{v_1^2}{2g}$$

$$\Rightarrow h = v_1^2/2g - h_0$$

3) Compute period of small oscillations for a body floating on water with $\rho = 0.9 \rho_w$.



Equilibrium is reached when

$$0 = mg - \rho_w A x_0 g$$

$$\Rightarrow 0.9 \cancel{\rho_w} \cancel{A} h \cancel{g} = \cancel{\rho_w} \cancel{A} x_0 \cancel{g}$$

$$\Rightarrow x_0 = 0.9 h$$

By dipping it by a small amount,

$$ma = mg - \rho_w A (x_0 + x) g$$

$$ma = -\rho_w A x g$$

$$\Rightarrow 0.9 \cancel{\rho_w} \cancel{A} h \, a = -\cancel{\rho_w} \cancel{A} x g$$

$$\Rightarrow \ddot{x} = -\frac{g}{0.9h} x$$

$$\therefore \omega = \sqrt{g/0.9h} \quad \& \quad T = 2\pi \sqrt{0.9h/g}$$