Lecture 11

Thursday, 7 November 2019 2:18 PM

Todays class will be heavy with calculations as we want to learn to apply the concepts. The theory can be summarized rather quickly.

They go as

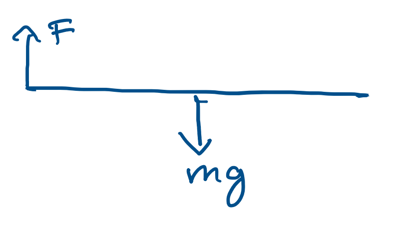
i) All points on a rigid body experience the same 'ω' & 'α'.
ii) $\tau = I\alpha = dL/dt$

$a = \alpha R$

iii) unconstrained bodies in free space rotate about their center of mass.

Lets start solving right away.

1) A uniform rod of length l and mass m rests on supports at its ends. The right support is quickly removed. What is the force on the left support immediately after?



$$mg - F = ma$$

$$F(l/2) = \left(\tfrac{1}{12} ml^2\right)\alpha$$

Immediatly after when the left end is still stationary.

$$\frac{l}{2}\alpha = a$$

Solving them we get $a = 3/4\, g$

$$\alpha = \frac{3}{2}\, g/l$$

$$F = mg/4$$

Alternatively, (for the moment after release)

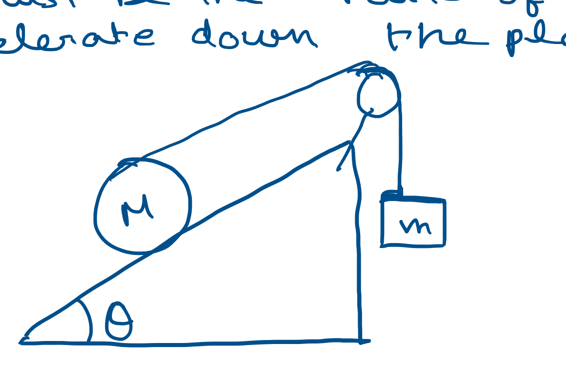
$$mg\,(l/2) = \left(\tfrac{1}{3} ml^2\right)\alpha \quad \left[\text{about left end}\right]$$

Using $F\,(l/2) = \left(\tfrac{1}{12} ml^2\right)\alpha$

$\therefore F = mg/4$

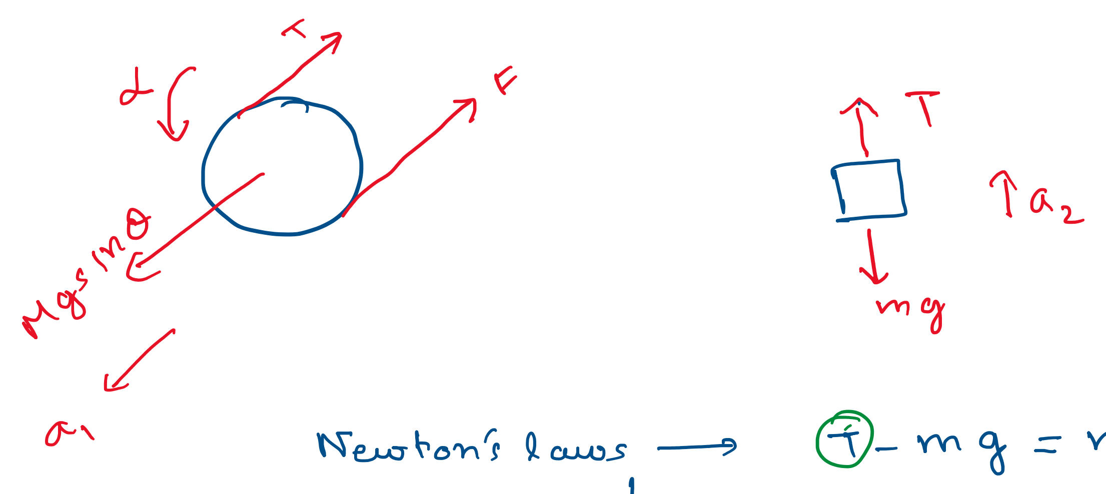
2) A string wraps around a uniform cylinder of mass M and radius 'R', which rests on a fixed plane at angle θ. The string passes over a massless pulley and is connected to a mass 'm'. Assume that the cylinder rolls without slipping on the plane, and that the string is parallel to the plane.

1) What is the acceleration of mass 'm'?
2) What must be the ratio of M/m for the cylinder to accelerate down the plane?



'Consider friction'

Method → i) Draw FBD ii) write equations



Unknowns

Newton's laws → $T - mg = m a_2$

↳ $Mg\sin\theta - T - F = M a_1$

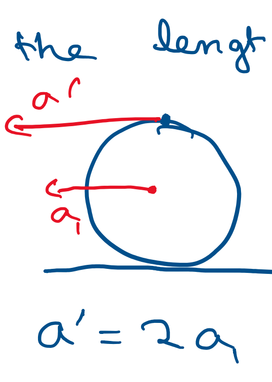
Balancing Torque

$$\underbrace{(F - T)}_{\text{note choice of signs}} R = \left(\tfrac{1}{2} MR^2\right)\alpha$$

Using slipping without rolling

$$a_1 = \alpha R$$

Constraing the length of string



$a' = 2a$ using $a = \alpha R$

$\therefore \quad x + y = l$

$\Rightarrow 2a_1 - a_2 = 0$

$\Rightarrow 2a_1 = a_2$

This gives us

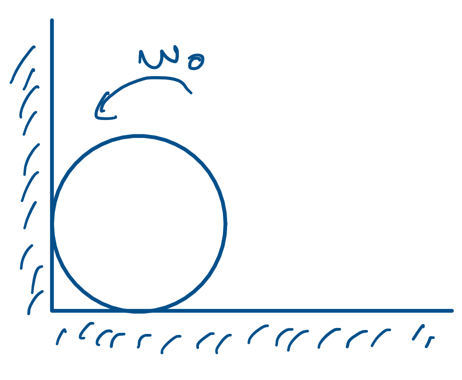
$$a_2 = \frac{4\,(Mg\sin\theta - 2mg)}{3M + 8m}$$

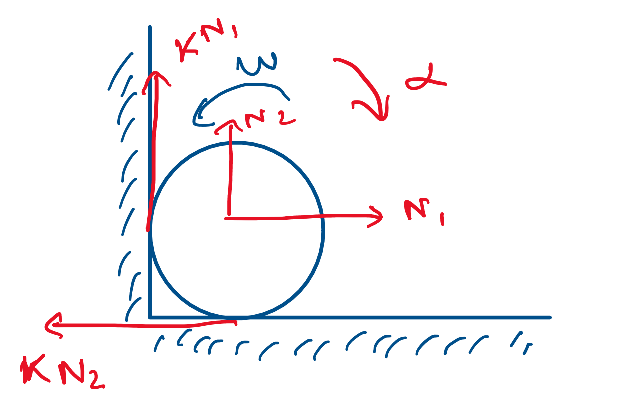
b) for the cylinder to accelerate down the plane,

$$M\sin\theta > 2m$$

$$M/m > 2/\sin\theta$$

3) A uniform cylinder of radius R is spinned about an axis to angular velocity ω_0 and then placed in a corner. The coefficient of friction between the corner walls and the cylinder is equal to 'k'. How many turns will the cylinder accompolish before it stops?





Newton's laws

$$kN_1 + N_2 = mg$$
$$kN_2 = N_1$$

Equating for Torque

$$kN_1R + kN_2R = \frac{mR^2}{2}\alpha$$

Solving for the above

$$\alpha = \frac{2kg\,(1+k)}{R\,(1+k^2)}$$

Now $\omega^2 = \omega_0^2 - 2\alpha\theta_s$

$$\Rightarrow \theta_s = \frac{\omega_0^2}{2\alpha} = \frac{\omega_0^2 R\,(1+k^2)}{4kg\,(1+k)}$$

$$\therefore \text{No of turns} = \theta/2\pi = \omega_0^2 R\,(1+k^2) \big/ 8\pi kg\,(1+k)$$

(essentially a pendulum problem) (we will see this in detail soon)

4) A disc with 'I_m' rotates in an horizontal plane. It is suspended by a thin massless rod. If the disk is rotated away from its equilibrium position by an angle θ, the rod exerts a restoring torque of '$b\theta$'. At $t=0$, the disk is released from rest and angular displacement θ_0. Find subsequent $\theta(t)$

$$\tau = -b\theta$$

$$I\alpha = -b\theta$$

$$\alpha = -\frac{b}{I}\theta \Rightarrow \ddot{\theta} = -b/I\,\theta$$

$$\therefore \theta(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$\omega_0 = \sqrt{b/I_m}$$

We will discuss this in details in the next class.