Lecture 1

Wednesday, 2 October 2019 9:20 PM

Welcome to the recitation sessions for Ph 1a. I am Raj and in this section we shall usually begin by reviewing some theory during the first half of the class and then proceed to apply them in problem. However, some days may have lots of theory while other days only a problem. Anyway, 9th will be mostly dealing the topic we are studying at that time.

Before we proceed further, let me give you a few essential info:

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Advice to you all: Ask Questions!!!

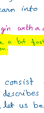
Additionally, it would be really awesome if you could fill these index cards with your name, pronouns, something you would like to learn in this course and something about you. While I can't promise I will try to include that something you want to learn into some of our lectures instead.

Let us begin with a quick math review on vectors:

I shall be a bit faster than the usual pace for this recitation.

Vectors:

Vectors consist of a magnitude & direction. What it describes is based on the units. However, let us begin with a more geometrical discussion.



$$\vec{A} = 4\hat{x} + 3\hat{y} = 4\hat{x} + 3\hat{x} = 4\hat{e}_x + 3\hat{e}_y$$
$$= \langle 4, 3\rangle \leftarrow \text{Nobody uses this anymore so avoid it}$$
$$= (4, 3)$$

What this means for m, m/s etc.

In our world is true dimensional (at least for this course) we shall use $\hat{x}, \hat{y}, \hat{z}$ to describe our vectors but the essential idea remains the same.

Note: You do not need to use cartesian ones to describe vector. It is mostly convenient and anything else depends on your intuition. We will avoid that here and only discuss it though a problem in Ph 1b when studying SR.

The above is known as the cartesian system. It is not unique and we tend to use other systems depending on the problem.

For example, polar coordinate system uses the the length of the vector and its angle with the x-axis to describe a vector.

Let's look at our previous example.

$$\therefore \vec{A} = \text{radius}\sqrt{3^2+4^2}$$
$$\text{angle } \tan^{-1}(3/4)$$
$$= 5\,\hat{e}_r + \tan^{-1}(3/4)\,\hat{e}_\theta$$

or

$$\vec{A} = r\cos\theta\,\hat{x} + r\sin\theta\,\hat{y}$$

$$\boxed{r = \sqrt{x^2+y^2} \quad \& \quad x = r\cos\theta \quad \theta = \tan^{-1}(y/x) \quad y = r\sin\theta}$$

How implies that you should remember this

Since we are already discussing trigonometric functions the handout contains a list of trigonometric identities that you should remember.

Similarly, for 3D vectors.

$$\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$$

$$A_x = r\sin\theta\cos\phi$$
$$A_y = r\sin\theta\sin\phi$$
$$A_z = r\cos\theta$$
$$r = \sqrt{A_x^2 + A_y^2 + A_z^2}$$
$$\phi = \tan^{-1}(A_y/A_x)$$
$$\theta = \cos^{-1}\left(\frac{\sqrt{A_x^2+A_y^2}}{r}\right)$$

Spherical coordinates

Vector Algebra

$\vec{A} = A_1\hat{x} + A_2\hat{y}$

$\vec{B} = B_1\hat{x} + B_2\hat{y}$

(will use 2D vectors as I am really bad at drawing 3D vectors.)

$$\vec{A} + \vec{B} = (A_1 + B_1)\hat{x} + (A_2 + B_2)\hat{y}$$

Take this as a definition

$$c\vec{A} = cA_1\hat{x} + cA_2\hat{y}$$

Dot Product

$$\vec{A}\cdot\vec{B} = A_1B_1 + A_2B_2 \leftarrow \text{Take this as a definition}$$

This doesn't give us the full picture (but I recommend for a cleaner picture)

$$= r_1r_2\cos\theta_1\cos\theta_2 + r_1r_2\sin\theta_1\sin\theta_2$$
$$= r_1r_2\cos(\theta_1 - \theta_2)$$
$$= |\vec{A}||\vec{B}|\cos\theta$$

norm or length

angle between the vectors (demonstrate visually)

(Visually: $\vec{A}$ dot product with $\hat{B}$ (with a unit vector in the components $\hat{B}$ along the direction.)

$$\vec{A}\cdot\hat{x} = A_1$$
$$\vec{A}\cdot\hat{y} = A_2$$

Properties → scalar
→ commutative ($\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$)
→ distributive ($\vec{A}\cdot(\vec{B}+\vec{C}) = \vec{A}\cdot\vec{B} + \vec{A}\cdot\vec{C}$)
→ $\vec{A}\cdot\vec{B} = |\vec{A}|^2$
→ $\vec{A}_1\cdot\vec{A}=0$, then $\vec{A}\perp\vec{B}$

Problem

For a general 2D vector, $\vec{A} = A_1\hat{x} + A_2\hat{y}$, find $\vec{B}$ such that it has the same magnitude but rotated counter clockwise

There are two ways of doing this, we could take the dot product way.

Take $\vec{B} = B_1\hat{x} + B_2\hat{y}$

$$\vec{A}\cdot\vec{B} = 0 \quad \text{as they are at } 90° \text{ to each other}$$
$$A_1B_1 + A_2B_2 = 0$$
$$\Rightarrow B_2 = -\frac{A_1B_1}{A_2}$$

Also, $|\vec{B}| = |\vec{A}|$

$$B_1^2 + B_2^2 = A_1^2 + A_2^2$$
$$\Rightarrow B_1^2 + \left(\frac{A_1B_1}{A_2}\right)^2 = A_1^2 + A_2^2$$
$$\Rightarrow B_1 = \pm A_2$$
$$B_2 = \mp A_1$$

There are two possible solutions, one rotated clockwise and other rotated counter-clockwise.

Take $A_2 = 0$ to find the right one.

$$\vec{B} = (-A_2, A_1)$$

Cross Product

(I will avoid discussing too much of its interpretation here. We will look at it more deeply when discussing rotations)

$$\vec{A}\times\vec{B} = (A_1B_2 - A_2B_1)\hat{z} + (A_2B_3 - A_3B_2)\hat{y}$$
$$+ (A_3B_1 - A_1B_3)\hat{x}$$
$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$$

Proof

$$|\vec{A}\times\vec{B}|^2 = (A_1B_2 - A_2B_1)^2 + (A_2B_3 - A_3B_2)^2$$
$$+ (A_3B_1 - A_1B_3)^2$$
$$= A_1^2(B_2^2 + B_3^2) + A_2^2(B_1^2 + B_3^2)$$
$$+ A_3^2(B_1^2 + B_2^2) - 2A_1A_2B_1B_2$$
$$+ A_2A_3B_2B_3 + A_1A_3B_1B_3$$

$$(\vec{A}\cdot\vec{B})^2 = A_1^2B_1^2 + A_2^2B_2^2 + A_3^2B_3^2$$
$$+ 2(A_1B_1A_2B_2 + A_2A_3B_2B_3 + A_1A_3B_1B_3)$$

$$|\vec{A}\times\vec{B}|^2 + (\vec{A}\cdot\vec{B})^2 = |\vec{A}|^2|\vec{B}|^2$$
$$\therefore |\vec{A}\times\vec{B}|^2 = |\vec{A}|^2|\vec{B}|^2 - (\vec{A}\cdot\vec{B})^2$$
$$= |\vec{A}|^2|\vec{B}|^2 - |\vec{A}||\vec{B}|^2\cos^2\theta$$
$$= |\vec{A}||\vec{B}|^2\sin\theta$$

Properties → vector
→ anti-commutative ($\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$)
→ Not associative ($\vec{A}\times(\vec{B}\times\vec{C}) \neq (\vec{A}\times\vec{B})\times\vec{C}$)
(Cross $\hat{x}\times\hat{y}$, $\hat{y}\times\hat{z}$, $\hat{z}\times\hat{x}$)
→ Distributive over addition
$\vec{A}\times(\vec{B}+\vec{C}) = \vec{A}\times\vec{B} + \vec{A}\times\vec{C}$
→ $c(\vec{A}\times\vec{B}) = (c\vec{A})\times\vec{B} = \vec{A}\times(c\vec{B})$
→ $\vec{A}\times\vec{B} = \vec{A}$ is zero if $\vec{A}$ is parallel to $\vec{B}$
→ $\vec{A}\times\vec{B}$ is orthogonal to both $\vec{A}$ & $\vec{B}$

Infinitesimals

(I shall take a slightly different approach with the section that I did experimenting with)

1D

2D

Find the area of a circle of radius R

$$A = \int r\,dr\,d\theta$$
$$= \pi R^2$$

3D