Formula Sheet 4

Saturday, 7 December 2019 6:23 PM

⊛ Angular Momentum for extended body

$$\vec{L} = M(\vec{R}_{CM} \times \vec{V}_{CM}) + I_{CM}\,\omega_{CM}\,\hat{z}$$

- $\vec{R}_{CM}$: position of CM
- $\vec{V}_{CM}$: velocity of CM
- I_{CM}: Moment of inertia about axis through CM
- ω_{CM}: ω about axis passing through CM

⊛ KE of extended rotating body

$$KE = \tfrac{1}{2} M V_{CM}^2 + \tfrac{1}{2} I_{CM}\,\omega_{CM}^2$$

- V_{CM}: velocity of CM
- ω_{CM}: ω about CM axis

⊛ $\perp$ axis theorem

$$I_z = I_x + I_y$$

(only holds for laminar bodies)

- I_x, I_y: passes through body & CM
- I_z: pass $\perp$ to the body and through CM

⊛ $\parallel$ axis theorem

$$I = I_{CM} + Md^2$$

- M: Mass of body
- d: distance from CM to new axis
- I_{CM}: I about CM
- I: I about new axis

⊛ $\tau = I\alpha$

⊛ In rolling without slipping

$$v = \omega R$$

$$a = \alpha R$$

⊛ For SHM, show

$$m\ddot{x} = -kx \quad \text{or} \quad \tfrac{1}{2}kA^2 = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

$$\omega = \sqrt{k/m} \quad \& \quad T = 2\pi/\omega$$

All x that follow the above relation,

$$x(t) = A\cos(\omega t + \delta)$$

A & δ obtained using initial conditions

To solve SHM problems,

1) Find eq condition.
2) Displace slightly
3) Write new eq of motion

1) Write E
2) Equate $dE/dt = 0$

⊛ If $m\ddot{x} = -kx - \gamma\dot{x}$

$$\ddot{x} + \frac{\gamma}{m}\dot{x} + \frac{k}{m}x = 0$$

$$\Rightarrow \ddot{x} + \beta\dot{x} + \omega_0^2 x = 0$$

$$x(t) = Ce^{-\beta t/2}\cos\left(\sqrt{\omega_0^2 - \beta^2/4}\;t + \delta\right)$$

⊛ Potential energy in gravity $= -\dfrac{GMm}{r}$

⊛ In orbit, Energy and Angular Momentum is alway conserved.

$$E = \frac{1}{2}mv_r^2 + \frac{1}{2}mv_\theta^2 - \frac{GMm}{r}$$

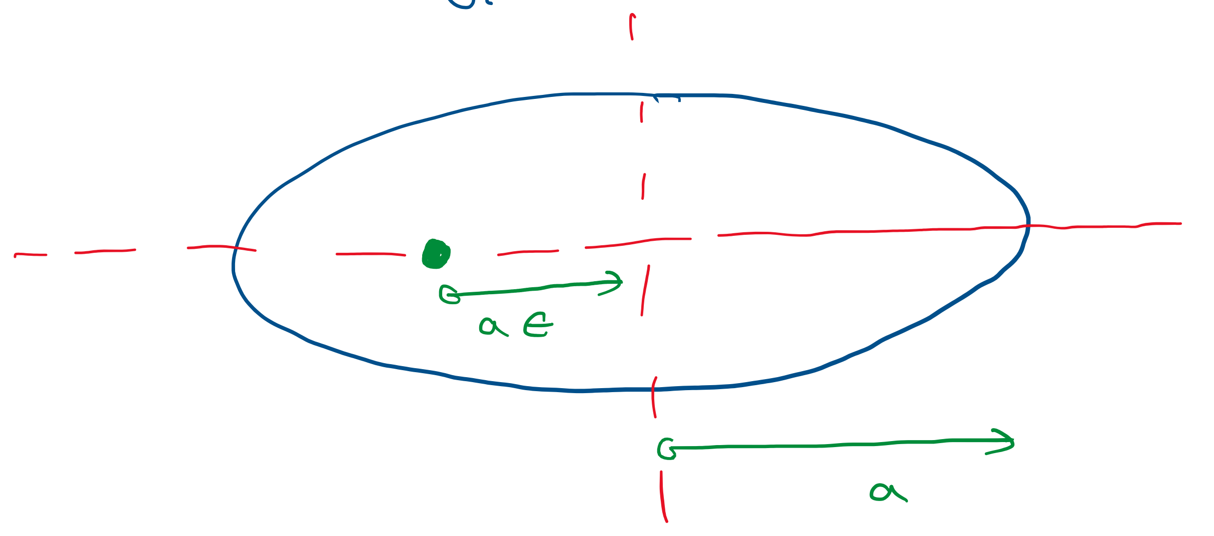
- v_r: radial velocity
- v_θ: tangential velocity

$$L = mv_\theta r$$

Equation of orbit

$$r = \frac{d}{(1 + \epsilon\cos\theta)}$$

$$d = \frac{L^2}{GMm^2} \quad \& \quad \epsilon = \left(1 + \frac{2EL^2}{G^2M^2m^3}\right)^{1/2}$$



$$a = d/(1-\epsilon^2)$$

$$r_{min} = \frac{d}{1+\epsilon} = a(1-\epsilon)$$

$$r_{max} = \frac{d}{1-\epsilon} = a(1+\epsilon)$$

For $E < 0$, it is bounded / elliptical orbit
$E > 0$, it is unbounded / hyperbolic orbit

⊛ $T^2 = \dfrac{4\pi^2}{GM}a^3$

$T^2 \propto a^3$

} Kepler's third law