Formula Sheet

Monday, 11 November 2019 10:37 AM

$r = \sqrt{x^2 + y^2}$, $\theta = \tan^{-1}(y/x)$ } for a point (x, y) in space.

$x = r\cos\theta$, $y = r\sin\theta$ } for a point (r, θ) in space

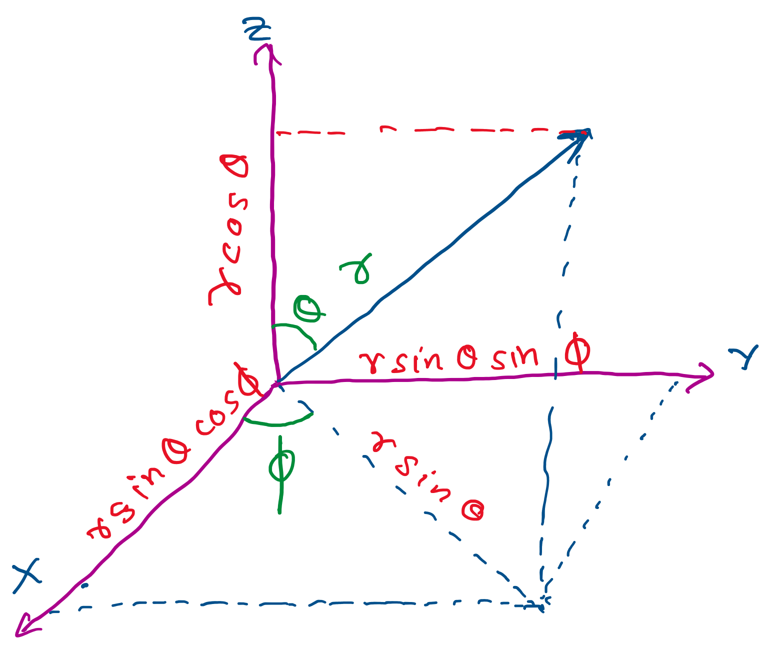
In 3D,

$x = r\sin\theta\cos\phi$
$y = r\sin\theta\sin\phi$
$z = r\cos\theta$

$r = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\phi = \tan^{-1}(y/x)$

$\theta = \tan^{-1}\left(\sqrt{x^2+y^2}/z\right)$



$\vec{A}\cdot\vec{B} = A_x B_x + A_y B_y + A_z B_z$
$= |A||B|\cos\theta$

$$\vec{A}\times\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = (A_y B_z - A_z B_y)\hat{i} + (A_z B_x - A_x B_z)\hat{j} + (A_x B_y - A_y B_x)\hat{k}$$

$|\vec{A}\times\vec{B}| = |\vec{A}||\vec{B}|\sin\theta$

Kinematics

A position in space is described by $\vec{r}$

$\vec{v} = d\vec{r}/dt$

$\vec{a} = d\vec{v}/dt$

For constant a,

$v = v_0 + at$

$x = x_0 + v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$

$\vec{V}_{AB} = \vec{V}_A - \vec{V}_B$

velocity of A as observed by B.

Newtonian Mechanics

Steps
- Draw the FBDs
- Write the equations of motion i.e. $F = ma$
- Write the constraints.
- Solve.

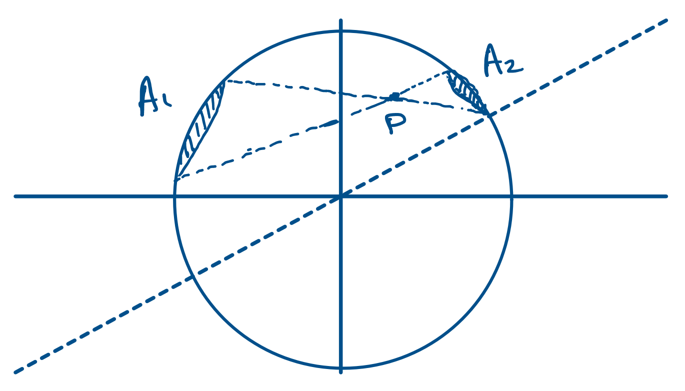
$\vec{F}_{ext} = M\vec{a}_{CM}$

this is the acceleration of center of mass.

Gravity

$$\vec{F}_G = -\frac{GMm}{r^2}\hat{r}$$

Shell theorem



Inside a mass shell, force due to shell is zero

Outside a mass shell, it acts as if the whole mass is at the center.

Fictitious Forces

force in inertial frame

$\vec{F}' = \vec{F} - m\vec{a}_0$ ← acceleration of non-inertial frame

mass of body being studied

force in non-inertial frame

In rotating frames,

measured in inertial frame

$$\vec{F}_{rot} = \vec{F}_{in} - m\left(2\vec{\omega}\times\vec{V}_{rot} + \vec{\omega}\times(\vec{\omega}\times\vec{r})\right)$$

velocity of mass in rotational frame

Energy

$$E_{mechanical} = \frac{1}{2}mv^2 - \int_{x_0}^{x} F(x)\,dx$$

reference point

Kinetic energy

Potential energy

$F_x = -\partial U/\partial x$

$\vec{F} = -\vec{\nabla}U$ where $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\Delta W = \int_1^2 \vec{F}\cdot d\vec{s} = \Delta K$ (Work Energy Theorem)

$dE/dt = \partial U/\partial t$

$\vec{\nabla}\times\vec{F} = 0$ (Conservative forces)

Momentum

$\vec{p} = m\vec{v}$

$R_{CM} = \frac{\sum m_i \vec{r}_i}{\sum m_i}$, $M V_{cm} = \sum m_i \vec{v}_i$, $M A_{CM} = \sum m_i \vec{a}_i$

Momentum is always conserved for $\vec{F}_{ext} = 0$

$$COR\ (e) = \frac{V'_{rel}}{V_{rel}}$$

V'_{rel} ← relative velocity of leaving

V_{rel} ← relative velocity of approach

Angular Momentum

$\vec{L} = \vec{r}\times\vec{p}$ ← for a particle in motion

$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r}\times\vec{F}$

Angular momentum is alway conserved for $\vec{\tau}_{ext} = 0$

distance to the CM from point of measurement

velocity of CM

ω about center of mass

$$\vec{L} = M(\vec{R}_{CM}\times\vec{V}_{CM}) + I_{CM}\,\omega_{cm}\hat{z}$$

I about center of mass

for a freely moving and rotating body