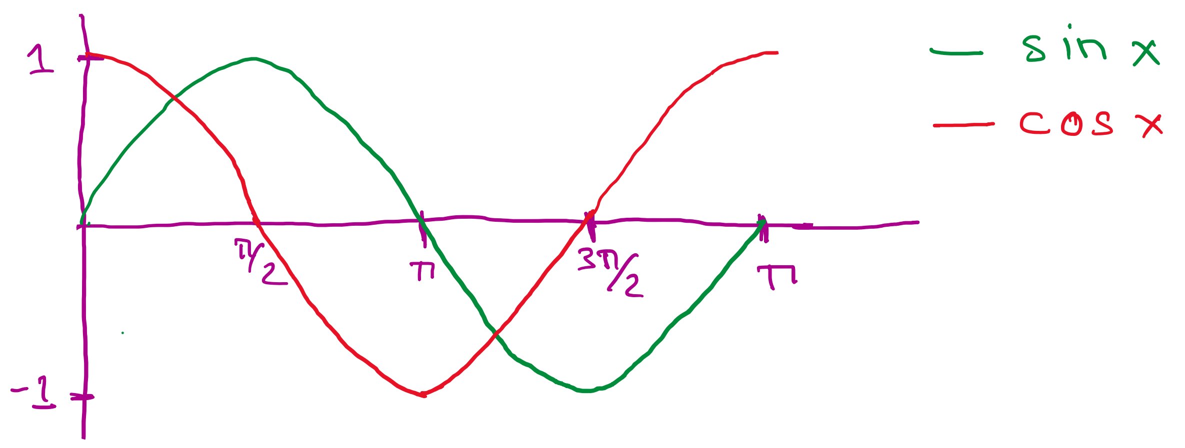
Handout 1

Thursday, 3 October 2019 9:38 AM



$$\cos 0 = 1 \qquad \sin 0 = 0$$

$$\cos \pi/6 = \sqrt{3}/2 \qquad \sin \pi/6 = 1/2$$

$$\cos \pi/4 = 1/\sqrt{2} \qquad \sin \pi/4 = 1/\sqrt{2}$$

$$\cos \pi/3 = 1/2 \qquad \sin \pi/3 = \sqrt{3}/2$$

$$\cos \pi/2 = 0 \qquad \sin \pi/2 = 1$$

Also,

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

$$\cos(2x) = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\sin(\pi - x) = \sin x$$

$$\sin(\pi/2 - x) = \cos x$$

$$\sin(-x) = -\sin x$$

$$\cos(\pi - x) = -\cos x$$

$$\cos(\pi/2 - x) = \sin x$$

$$\cos(-x) = \cos x$$

Taylor Series

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \ldots$$

Important Taylor expansions

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots = \sum_{j=0}^{\infty} (-1)^j \frac{x^{2j}}{(2j)!}$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{4!} - \ldots = \sum_{j=0}^{\infty} (-1)^j \frac{x^{2j+1}}{(2j+1)!}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{j=0}^{\infty} \frac{x^j}{j!}$$

(You can derive cos x & sin x expansion using e^x epansion & euler's formula)

Euler's formula

$$e^{ix} = \cos x + i \sin x$$

$$\therefore \cos x = \frac{e^{ix} + e^{-ix}}{2}$$

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i}$$