Quantum Precision Limits of Displacement Noise Free Interferometers

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arXiv Link

Abstract

Current laser-interferometric gravitational wave detectors suffer from a fundamental limit to their precision due to the displacement noise of optical elements contributed by various sources. Several schemes for Displacement-Noise Free Interferometers (DFI) have been proposed to mitigate their effects. The idea behind these schemes is similar to decoherence-free subspaces in quantum sensing i.e. certain modes contain information about the gravitational waves but are insensitive to the displacement noise. In the associated paper we derive quantum precision limits for general DFI schemes, including optimal measurement basis and optimal squeezing schemes. We introduce a triangular cavity DFI scheme and apply our general bounds to it. Precision analysis of this scheme with different noise models shows that the DFI property leads to interesting sensitivity profiles and improved precision due to noise mitigation and larger gain from squeezing. Further extensions of this scheme are also discussed in the manuscript.

What is Displacement-Noise Free Interferometry (DFI)?

- The idea is to find sub-spaces that contain significant levels of the signal while being unaffected by the noise.
- We use techniques of quantum metrology to derive general precision limits, optimal measurements and optimal squeezing quadratures that are most resistant to displacement noise.

Formalism

We use the input-output formalism given by:

$$\vec{b}\left(\Omega\right)=M\left(\Omega\right)\vec{a}\left(\Omega\right)+\mathcal{V}\left(\Omega\right)\vec{h}\left(\Omega\right)+A\left(\Omega\right)\vec{\Delta x}\left(\Omega\right)$$

where $M\left(\Omega\right),\ A\left(\Omega\right),\ \mathcal{V}\left(\Omega\right)$ are the transfer matrices of the input modes, displacement noise, and the GW vector respectively. It can be shown that the QFIM in such a scenario for Gaussian modes is given by:

$$I=2\left(\partial_{\vec{h}}\vec{d}_q\right)^{\dagger}\Sigma_q^{-1}\left(\partial_{\vec{h}}\vec{d}_q\right)$$

$$\implies I=4\mathcal{V}^{\dagger}\left(MM^{\dagger}+\delta^2AA^{\dagger}\right)^{-1}\mathcal{V}$$

Primary Results

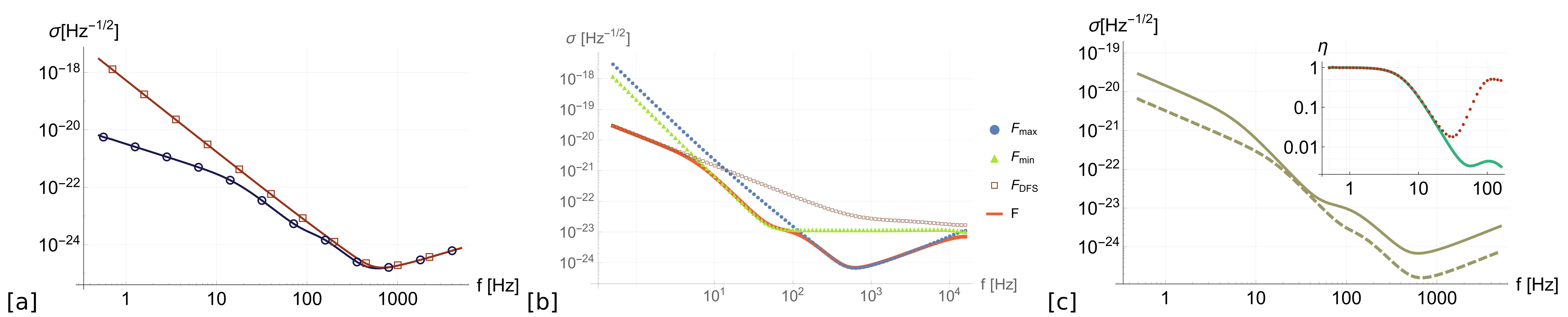

Figure 1. *(a) Comparison between the QFI with squeezing (blue curve, circles) and the FI with maximum signal combination (red curve, squares). (b) Given thermal displacement and radiation pressure noise with phase quadratures measurement there are three different regimes of the FI: the solid red line correspond to F, the blue dots to F_{max}, green triangles to F_{min} and brown squares to F_{DFS}. The different FI's coincide with total FI in three different regimes. (c) Performance with squeezing given thermal and radiation pressure noises. The solid (dashed) line corresponds to unsqueezed (optimally squeezed) std with phase quadratures measurement. Inset: η_{gain} (red dots) and η (green line) as a function of frequency.*

Scheme

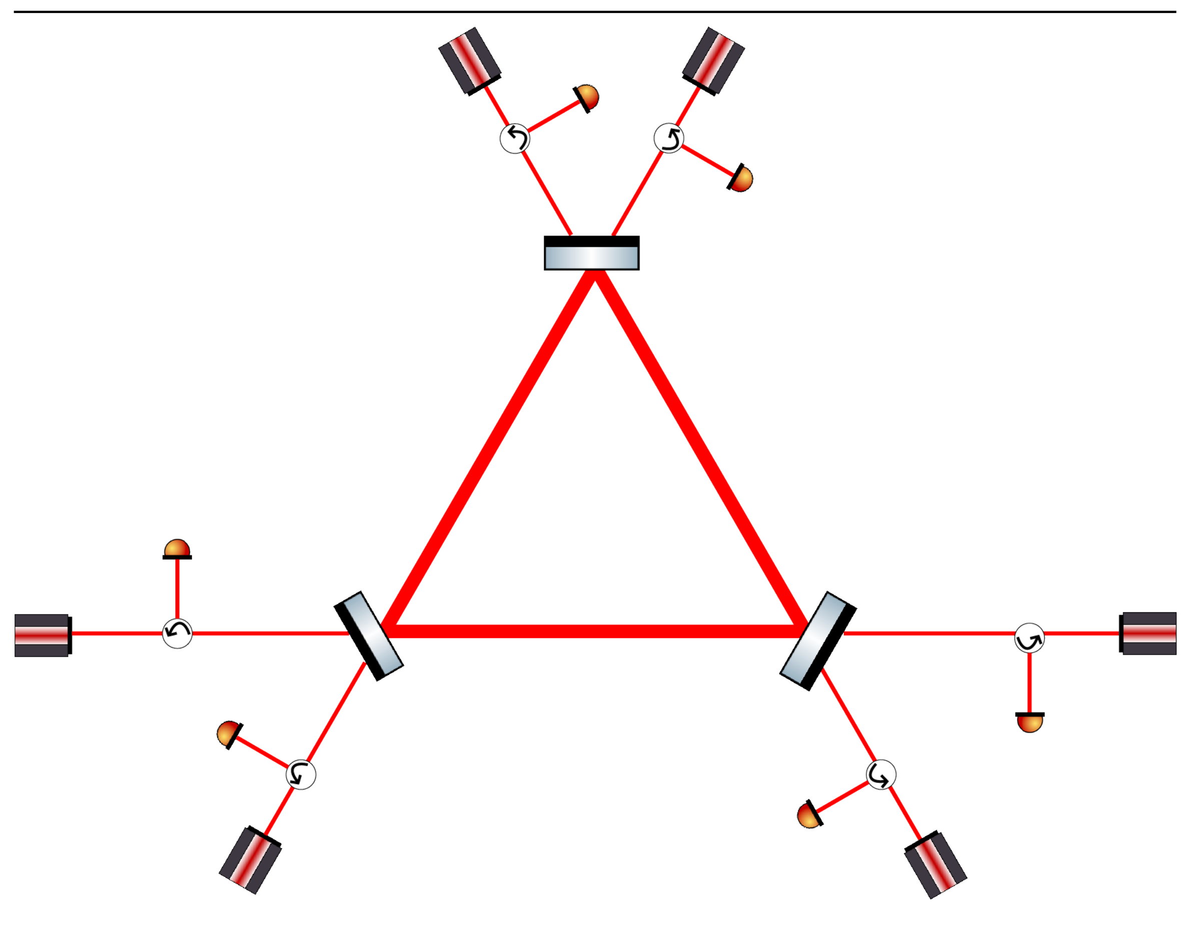

Figure 2. *A triangular cavity is formed by three mirrors. Three clockwise circulating input and output fields exist; alongside three anti-clockwise input and output fields.*

Gain

$$\eta=\frac{F_{\mathrm{DFS}}}{F},$$

where F_{DFS} is the FI coming from the DFS.

$$\eta_{\mathrm{gain}}=\frac{\frac{F_{\mathrm{sq}}}{F}-1}{e^{2s}-1},$$

where $F_{\mathrm{sq}}\left(F\right)$ is the FI with(out) squeezing. We demonstrate

$$\eta_{\mathrm{gain}}=\eta$$

Optimal Measurements

Given squeezing of phase quadratures, the distributed quadrature that saturates QFI bound is:

$$\begin{pmatrix}-M_{\mathrm{int}}M_{21}^{\dagger}\\ 1\end{pmatrix}\left(e^{-2s}\mathbb{1}+\delta^2A_{\mathrm{ph}}A_{\mathrm{ph}}^{\dagger}\right)^{-1}\mathcal{V}_{\mathrm{ph}}$$

Symmetrically, given phase quadrature measurement, optimal squeezing (that saturates the QFI bound) is given by,

$$\begin{pmatrix}M_{\mathrm{int}}^{\dagger}M_{21}\\ 1\end{pmatrix}\left(1+\left(M_{\mathrm{int}}^{\dagger}M_{21}\right)^2\right)^{-1/2}$$

Acknowledgements

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